Your Name: ______________________________

Circle your TA's name:

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Mathematics 221, Fall 2007 **Lecture 1 (Wilson)**

First Midterm Exam October 11, 2007

There are some trig function values on the back page that you might find useful.

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on an index card, as announced in class and at the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	9	
2	10	
3	14	
4	16	
5	13	
6	14	
7	12	
8	12	
TOTAL	100	

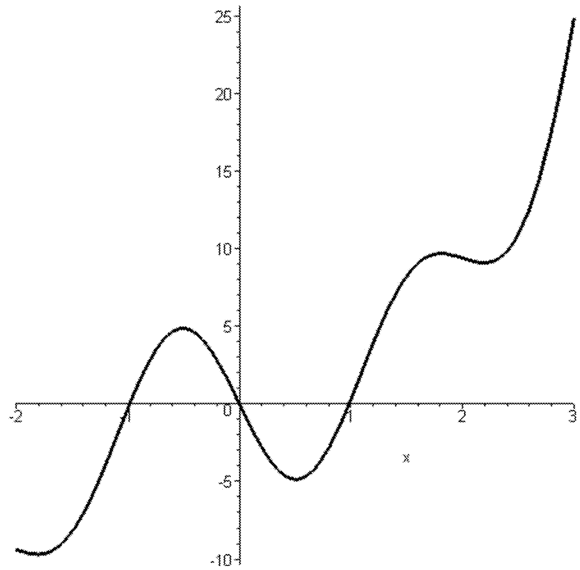
Problem 1 (9 points)
An object moves along the $y-$axis so that its position at time t (in seconds) is given by $y = t^3 - 2t + 1$ (in feet).

(a) What is its displacement (net distance moved) from $t = 0$ to $t = 2$?

(b) What is its average velocity from $t = 0$ to $t = 2$? (include units)

(c) What is its instantaneous velocity at $t = 2$? (include units)

Problem 2 (10 points)



For the function $y = f(x)$ graphed to the right, answer (a)-(e) as true (T) or false (F), and supply a numeric answer for (f) and (g).

(a) There is a number c between 1 and 2 such that $f'(c) = 0$. ________

(b) There is a number c between 1 and 2 such that $f(c) = 0$. ________

(c) If $2\frac{1}{2} < x < 3$, $f'(x) \geq 0$. ________

(d) The average rate of change of $f(x)$ from $x = -1$ to $x = 1$ is greater than 2. ________

(e) As x goes from -1 to 0, the derivative $f'(x)$ is decreasing. ________

(f) At approximately what value of x (for $x \in [-2, 3]$) does the derivative $f'(x)$ take its largest value? ________________

(g) At approximately what value of x (for $x \in [-2, 3]$) does the derivative $f'(x)$ take its smallest (most negative) value? ________________

Problem 3 (14 points)

(a) Evaluate: $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

Justify each statement or calculation. You do not need to use an $\epsilon - \delta$ argument, but you should refer to things like theorems and limit rules proved in lecture and in the textbook.

You will receive no credit for an unjustified answer.

(b) It is true that $\lim_{x \to 4} (-2x + 9) = 1$.

Use the $\epsilon - \delta$ definition of the limit to confirm this.

Problem 4 (16 points)

(a) Find the derivative of $y = \frac{\sin(x)}{x^2}$.

(b) Find the derivative of $y = x^3 \cos(3x - 2)$.

(c) Find the derivative of $y = (\sin(3x))^{100}$.

(d) Find the third derivative of $y = \sin(2x)$.

Problem 5 (14 points)
Find an equation for the tangent line to the graph of

$$x \sin(2y) = y \cos(2x)$$

at the point $(\frac{\pi}{4}, \frac{\pi}{2})$.

Problem 6 (12 points)

(a) Evaluate the limit. You must justify your answer, but you do not need to use precise arguments involving δ or ϵ.

$$\lim_{x \to 0} \frac{3x + \sin(x)}{4x}$$

(b) Use the definition of the derivative as a limit to find the derivative of $3x^2 - 2x + 5$.

Problem 7 (12 points)

(a) The radius of a sphere is growing at a rate of 3 inches per second.

How fast is the volume of the sphere changing at the instant when the radius is 10 inches?

(A formula for the volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Remember to include appropriate units.)

(b) The function defined by $f(x) = \frac{3x^2-7x-20}{x-4}$ is not continuous at $x = 4$. Construct a new function $g(x)$ which gives the same value for any $x \neq 4$ but such that $g(x)$ is continuous at $x = 4$.

Problem 8 (12 points)

(a) Find the linearization of $f(x) = \sin(x)$ at $x = \pi$.

(b) Use that linearization to estimate $\sin(\pi + 0.1)$.

(Just giving the value your calculator produces for $\sin(\pi + 0.1)$ will get no credit.)

Some values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	undefined
π	0	-1	0

SCRATCH PAPER