

Your Name: _____

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Mathematics 221, Fall 2003

Lecture 2 (Wilson)

First Midterm Exam October 7, 2003

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to make clear what is your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using $\frac{\pi}{3}$, $\sqrt{3}$, $\cos(0.6)$, and similar numbers) rather than using decimal approximations. If you use a calculator to evaluate your answer be sure to show what you were evaluating!

There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on an index card, as announced in class and at the class website.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" (without more details) are not sufficient substantiation...)

Problem	Points	Score
1	14	
2	12	
3	12	
4	12	
5	12	
6	14	
7	12	
8	12	
TOTAL	100	

Problem 1 (14 points)

(a) Find the derivative $f'(x)$ for $f(x) = \frac{x^3+2x+1}{x^2-3}$.

(b) Find the derivative $f'(x)$ for $f(x) = \sqrt{x^3 - 2x + 1}$.

(c) Find the second derivative $f''(x)$ for $f(x) = \cos(x^2 + 1)$.

Problem 2 (12 points)

Use the definition of the derivative as a limit to find $f'(x)$, for $f(x) = 2x^2 + x$.
(Just writing down $f'(x)$ from a formula will receive zero credit.)

Problem 3 (12 points)

Road N goes north and south, and road E goes east and west. A car is travelling north on N at 50 miles per hour. Another car is travelling east on E at 60 miles per hour. How fast is the distance between them changing at the instant when the first car is 20 miles south of where the roads meet and the second car is 15 miles east of where the roads meet? Is the distance between the cars increasing or decreasing at that moment?

Problem 4 (12 points)

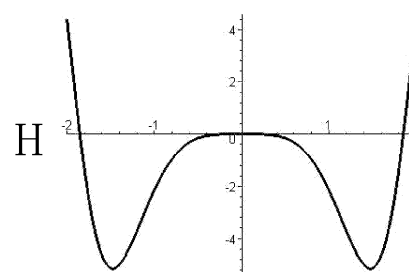
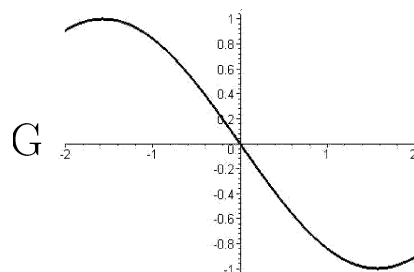
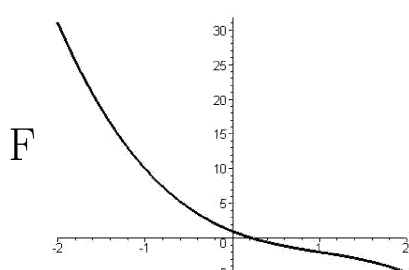
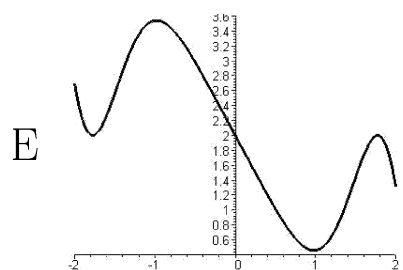
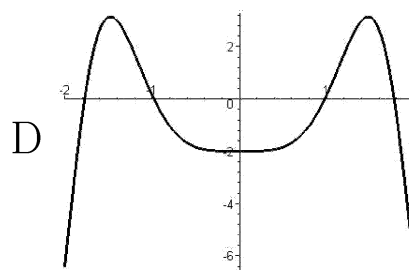
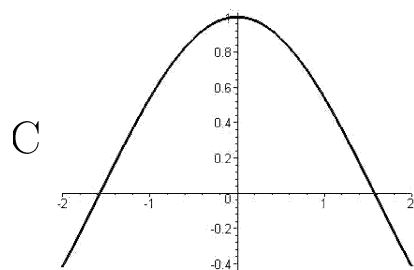
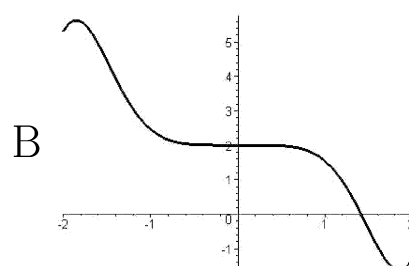
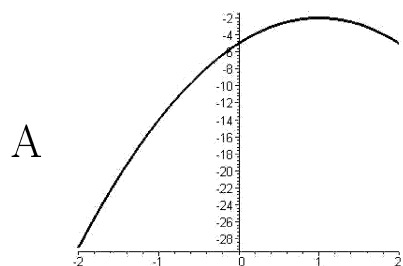
Of the eight functions graphed below, four are the derivatives of the other four. All are plotted with the same horizontal scale, but the vertical scales vary. Fill in the letters A-H in the blanks: each letter gets used just once.

Function _____ is the derivative of function _____.

Function _____ is the derivative of function _____.

Function _____ is the derivative of function _____.

Function _____ is the derivative of function _____.



Problem 5 (12 points)

$$\text{Let } f(x) = \frac{x^2 - 3x - 10}{x + 2}.$$

- (a) Then $f(x)$ is not continuous at $x = -2$. Why? Tell how it fails the definition of continuity at that point.

- (b) Now change the definition of $f(x)$ in such a way that your new version is continuous at $x = -2$. The new function should agree with the old one for most values of x .

Problem 6 (14 points)

Find the point (x_0, y_0) on the graph of $y = \frac{x}{x-3}$ (for $x > 3$) such that the tangent line to the graph at (x_0, y_0) passes through the point $(9, 1)$.

Problem 7 (12 points)

Use the definition of limit in terms of ϵ and δ to justify the statement

$$\lim_{x \rightarrow 4} (3x - 2) = 10.$$

You should both show what to use for δ and also demonstrate that your choice of δ does what is needed.

Problem 8 (12 points)

A point on a rotating bicycle tire goes up and down, and its height at time t (in seconds), is given by $y = 26 \sin t$ (in inches). (We ignore its horizontal motion in this problem.)

(a) What is the average velocity of the point on the tire, between the times $t = 0$ and $t = \pi$?

(b) What is the instantaneous velocity of the point at the instant when $t = \frac{\pi}{4}$?



SCRATCH PAPER