Mathematics 213, Section 4 (Wilson)

Your Name: _____

Final Examination 5/12/99

Write your answers to the twelve problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

You may refer to notes you have brought in, as announced in class.

There is a problem on the back of this sheet. Be sure not to skip over it accidentally!

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. IF YOU USE A CALCULATOR TO DO A SIGNIFICANT PART OF THE WORK ON A PROBLEM, WRITE OUT AN EXPLANATION OF JUST WHAT YOU ASKED THE CALCULATOR TO DO.

Problem	Points	Score
1	16	
2	16	
3	16	
4	16	
5	18	
6	18	
7	16	
8	16	
9	18	
10	18	
11	16	
12	16	
TOTAL	200	

Problem 1 (16 points)
Find the Taylor polynomial of degree 5 for x near 0, for

$$f(x) = \sin(x) - \cos(x).$$

Problem 2 (16 points)
A snail crawls in a straight line. His speed is measured every 10 minutes. He starts out quickly (for a snail) and never speeds up as he moves along. Here are the measured speeds:

Elapsed time (minutes)	0	10	20	30	40	50	60
Speed (feet/minute)	5	4	4	3	3	3	2

Give upper and lower estimates for the distance the snail travelled in the 60 minutes of observations. Explain your reasoning!

Problem 3 (16 points)
For the differential equation

$$\frac{dy}{dx} - 2y = 0 :$$

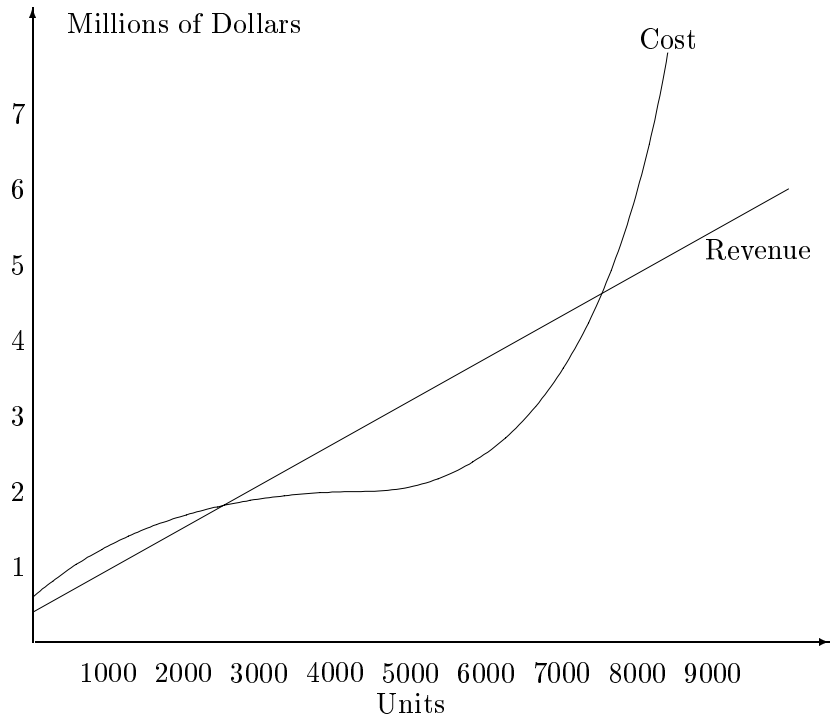
(a) Find all solutions $y(x)$.

(b) Find the particular solution satisfying

$$y(1) = \frac{e^2}{3}.$$

Problem 4 (16 points)

The picture shows cost and revenue for different production quantities for a product. Refer to the picture to do the problems below.



(a) Approximately what is the fixed cost of production, in millions of dollars?

(b) What is the smallest number of units to produce which does not lose money?

(c) How many units should be produced to obtain maximum profit? What is the profit (in millions of dollars) associated with that level of production?

(d) How many units should be produced to minimize the average cost of producing each unit? What profit (in millions of dollars) goes with that level of production?

Problem 5 (18 points)

Let $f(x, y) = 5 - x^2 - 2y^2$ and $g(x, y) = 6x + 8y$.

(a) What values for x and y maximize $f(x, y)$ subject to the constraint $g(x, y) = 17$?

(b) What is the maximum value of $f(x, y)$ subject to the constraint $g(x, y) = 17$?

(c) Would the maximum value of f increase or decrease if the constraint were changed to $g(x, y) = 16$? By about how much would it increase or decrease?

Problem 6 (18 points)

A model of interacting populations of robins (population $r(t)$ in thousands of robins at time t) and worms (population $w(t)$ in millions of worms at time t) has

$$\frac{dw}{dt} = 2w - w\,r$$

and

$$\frac{dr}{dt} = -2r + w\,r$$

(Note that this is similar to the model in the book but not quite the same...)
The figure to the right shows the slope field in the phase plane for this model, together with a couple of solution graphs.