# Mathematics 213, Section 4 (Wilson)

### Your Name: _____

## Exam I    2/15/99

Write your answers to the six problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

There is scratch paper on the back of this sheet. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one sheet of paper (regular notebook or typing size) as announced in class.

If you have to evaluate an integral and the problem does not say otherwise, you may either use a sum to approximate the integral or use an antiderivative and the fundamental theorem of calculus. If you use a sum, be sure to tell what you are doing: How many intervals? What values of the function?

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. IF YOU USE A CALCULATOR TO DO A SIGNIFICANT PART OF THE WORK ON A PROBLEM, WRITE OUT AN EXPLANATION OF JUST WHAT YOU ASKED THE CALCULATOR TO DO.

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 16 | |
| 2 | 17 | |
| 3 | 17 | |
| 4 | 16 | |
| 5 | 17 | |
| 6 | 17 | |
| TOTAL | 100 | |

# SCRATCH PAPER

Problem 1     (16 points)

Find the derivatives of:

(a)   $f(x) = 3x^2 - \cos(x)$

(b)   $g(x) = 3x - \ln(e^{2x} + x^2)$

Problem 2     (17 points)

In producing a certain product,

- The fixed costs are $500.

- The marginal cost is $C'(q) = q^2 - 50q + 700$ in dollars, if $q$ items are produced.

What is the total cost to produce 50 items?

Problem 3     (17 points)

Suppose that

$$q = 1000\,p^2\,e^{-.1p}$$

gives the number of units $q$ you will sell of a particular product, selling it at price $p$.

(a)   How many units will you sell if the price is $20?

(b)   What is the derivative of $q$ at $p = 20$? What does this tell you about demand for the product?
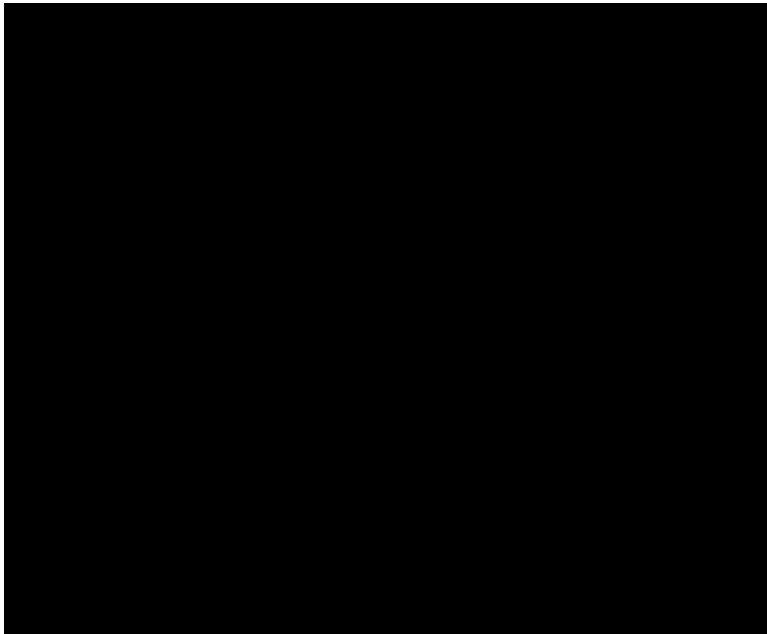
Problem 4    (16 points)

Part of the graph of a function $f(x)$ is shown below. Using that graph, estimate:

(a)   The integral of $f$ from $x = 0$ to $x = 1.5$

(b)   The area between the graph of $f$ and the $x$-axis, between $x = 0$ and $x = 1.5$.

Be sure to explain how you get your answers!

Problem 5     (17 points)

A function takes on the following values:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 9 | 8 | 9 | 12 | 17 |

(a)   Estimate $f'(2.5)$.

(b)   Estimate $f(2.5)$.

(c)   Use your results from (a) and (b) to estimate an equation for the tangent line to the graph of $f$ at $x = 2.5$.

(d)   Use your answer to (c) to estimate the value of $f(2.8)$.

Problem 6      (17 points)

On the axes below, draw a graph of a function $f(x)$ which has the following properties:

1. $f'(x) < 0$ for $x < -1$.

2. $f'(x) \geq 0$ for $-1 \leq x \leq 4$.

3. $f'(x) < 0$ for $4 < x$.

4. $f''(x) > 0$ for $x < 2$.

5. $f''(x) < 0$ for $x > 2$.

6. $f(1) = 0$.

Be sure to label the axis markings so as to show the scale.