Your Name: \_\_\_\_\_

Circle your TA's name:

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Mathemat	ics 211, Lecture	2 (Wilson)
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There are <u>ten</u> problems on this exam. Be sure not to overlook the one on the back of this sheet.

Write your answers to the problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

There is scratch paper at the end of the exam. If you need more scratch paper please ask. Answers you write on scratch paper will NOT be graded unless AT THE PROB-LEM you tell us where to find the answer!

Wherever applicable, leave your answers in exact forms (using  $\pi$ , e,  $\sqrt{3}$ ,  $\ln(2)$ , and similar numbers) rather than using decimal approximations.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	20	
2	26	
3	18	
4	20	
5	<b>24</b>	
6	20	
7	18	
8	16	
9	20	
10	18	
TOTAL	200	

Problem 1 (20 points)

An object moves up and down so that its height (in feet) above the ground at any time t (in seconds) is given by  $12t - 2t^2 + 14$ .

(a) How fast is the object moving when t = 1? Is it moving upward or downward?

(b) At what time t is the object at its highest?

(c) How high does the object get?

(d) After the object passes its highest point, when does it hit the ground?

Problem 2 (26 points) (a) (5 points) Find  $\frac{df}{dx}$  for  $f(x) = (e^{3x})\cos(2x)$ .

(b) (5 points) Find  $\frac{df}{dx}$  for  $f(x) = \frac{(x^2-2)\sqrt{x}}{3x-1}$ .

(c) (8 points) If  $f(x) = e^{3x-6}$ , find an equation for the line tangent to the graph of f(x) at the point (2,1).

(d) (8 points) For  $f(x, y) = \cos(x \ y)$  find: (i)  $\frac{\partial f}{\partial x}$ 

(ii)  $\frac{\partial^2 f}{\partial y \ \partial x}$ 

Problem 3 (18 points)

Let  $f(x) = 2x^3 - 12x^2 - 30x + 7$ .

- (a) For what values of x is f increasing?
- (b) For what values of x is f decreasing?
- (c) For what values of x is f concave upward?
- (d) For what values of x is f concave downward?

Problem 4 (20 points)

The population P(t) (at time t in years since observations were started) of a certain species is changing so that  $\frac{dP}{dt}$  is proportional to P(t). When first observed the population was 16,000. Six years later it is down to 2000.

(a) Write a formula expressing P(t) for all t.

(This formula will have the variable t in it but should have no other variables.)

(b) When is the population 4000?

(c) What is the population 15 years after observations start?

(d) If we assume conditions before observations started were the same as later, what was the population four years before the start of observations?

Problem 5 (24 points)

Evaluate the integrals: (a)

$$\int x \ e^{x^2} dx$$

(b)

$$\int x \ e^x dx$$

(c) 
$$\int_{\pi}^{2\pi} \left(\cos^2 \frac{x}{2}\right) \left(\sin \frac{x}{2}\right) dx$$

Problem 6 (20 points)

Find all relative maxima, relative minima, and saddle points of

$$f(x,y) = x^{3} - 6x^{2} + 9x + 2y^{3} - 3y^{2} - 12y - 1.$$

(For a maximum or minimum you should provide the value of the function as well as the point (x, y). For each point (x, y) which is critical be sure to tell <u>which</u> of maximum, minimum, or saddle point applies to that point.)

Problem 7 (18 points)

(a) (10 points) If x and y satisfy  $e^x \sin(y) = x$ , what is the value of  $\frac{dy}{dx}$  at the point (0,0)?

(b) (8 points) If  $f(x) = \sin(\ln(3x+4))$  what is f'(-1)? Problem 8 (16 points)

The graphs of  $y = \sin(x)$  and  $y = \cos(x)$  cross when  $x = \frac{\pi}{4}$  and when  $x = \frac{5\pi}{4}$  but not in between. What is the area between these two curves for  $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$ ?

Problem 9 (20 points)

Find the maximum and minimum values of f(x, y) = xy subject to the constraint  $x^2 + y^2 - 18 = 0$ .

Problem 10 (18 points)

What is the average value of the function  $f(x) = x \sin(x)$  on the interval  $[0, \pi]$ ?

## SCRATCH PAPER