Mathematics 211, Lecture 1 (Wilson) Circle your TA's name: Your Name: \_

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Exam III 5/4/93

Write your answers to the seven problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using  $\pi$ , e,  $\sqrt{3}$ , ln(2), and similar numbers) rather than using decimal approximations. Specifically: If you need the value of a trigonometric function such as  $\sin(\frac{\pi}{3})$  and if you know the answer exactly (in this case  $\frac{\sqrt{3}}{2}$ ) feel free to use that to simplify things. Otherwise you are better off to leave it as  $\sin(\frac{\pi}{3})$  than to use an approximation such as 0.866.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it. You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	12	
3	15	
4	12	
5	13	
6	12	
7	16	
8	8	
TOTAL	100	

## Problem 1 (12 points) Find the derivatives of the following functions:

(a) 
$$f(x) = \tan(x^2 + 2x)$$

(b) 
$$f(x) = \frac{\cos(x)}{x}$$

(c) 
$$f(x) = e^{\sin(x)}$$

(d) 
$$f(x) = \sec(1 + \cos(x))$$

Problem 2 (12 points) Let  $f(x, y) = x^2y + 3x\sin(y)$ . Find:  $\frac{\partial f}{\partial x}$ 

 $\frac{\partial f}{\partial y}$ 

 $\frac{\partial^2 f}{\partial x^2}$ 

 $\frac{\partial^2 f}{\partial y^2}$ 

 $\frac{\partial^2 f}{\partial x \partial y}$ 

 $\frac{\partial^2 f}{\partial y \partial x}$ 

Problem 3 (15 points) Evaluate the following integrals: (a)

$$\int x^2 \cos(x^3) dx$$

(b)



(c)

 $\int e^x \sin(x) dx$ 

Problem 4 (12 points)

a. Find the derivative of  $f(x) = \tan(x) \sec(x)$ .

b. Evaluate the integral

$$\int \csc^2(2x) dx$$

c. Evaluate the integral

$$\int \tan(x) \sec(x) dx$$

Problem 5 (13 points)

Let  $f(x) = 2x^3 + 3x^2 + 12x + 2y^3 + 3y^2 + 4$ .

Find all critical points of f(x, y), and identify each as being a relative maximum, a relative minimum, or neither.

Problem 6 (12 points)

Find the maximum and minimum values of y - x subject to the constraint that  $x^2 + y^2 = 2$ .

Problem 7 (16 points) Let  $f(x) = e^{-2x}$ .

a. Find the Taylor polynomial of degree 4 expanded about a = 0 for f(x).

b. Find the Taylor polynomial of degree 2 expanded about a = 1 for f(x). (Remember that numbers like  $\frac{1}{e}$  or  $e^{-1}$  are preferred to numbers like .37) Problem 8 (8 points)

Let  $f(x) = \sin(2x)$ .

Suppose you need to find a polynomial approximation to f(x) which is within at worst .1 of giving f(x) for any x in the interval (-1,1), and you decide to use the Taylor polynomial  $P_n(x)$  of degree n for f(x) about a = 0. What degree, *i.e.* what value for n, do you need? Be sure to explain how you choose your answer.

## SCRATCH PAPER