

Circle your TA's name:

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## Exam II 4/8/93

Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.

Wherever applicable, leave your answers in exact forms (using  $\pi$ ,  $e$ ,  $\sqrt{3}$ ,  $\ln(2)$ , and similar numbers) rather than using decimal approximations.

There is scratch paper at the end of the exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought in on one 4" by 6" index card, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

| Problem | Points | Score |
|---------|--------|-------|
| 1       | 16     |       |
| 2       | 10     |       |
| 3       | 15     |       |
| 4       | 12     |       |
| 5       | 10     |       |
| 6       | 15     |       |
| 7       | 10     |       |
| 8       | 12     |       |
| TOTAL   | 100    |       |

Problem 1 (16 points)

Evaluate the following integrals:

(a)

$$\int \frac{4x - 3}{4x + 2} dx$$

(b)

$$\int x \ln(x) dx$$

(c)

$$\int_0^2 x \sqrt{3x^2 + 4} \, dx$$

(d)

$$\int_1^2 e^{(2x-2)} dx$$

Problem 2 (10 points)

Find  $\frac{dy}{dx}$  when  $x$  and  $y$  satisfy:

(a)

$$2xe^y - xy^2 = x^2 + 3$$

(b)

$$y = \frac{(x+2)(e^x)}{(x^2-3)(2x+4)}$$

Problem 3 (15 points)

Find the derivatives of the following functions:

(a)  $f(x) = \ln(x^2 + e^x)$

(b)  $f(x) = e^{x^2+3x}$

(c)  $f(x) = (2x + 1)^{x^2}$

(d)  $f(x) = \log_6(x^3 - 2x)$

(e)  $f(x) = e^{\ln(x^2)}$

Problem 4 (12 points)

(a) Find the average value of the function  $f(x) = 6x^2 - 3$  on the interval  $[0, 2]$ .

(b)  $F(x)$  is a function whose derivative is  $3x^2 + 1$ . If  $F(1)$  is 0, what is  $F(x)$  in general?

Problem 5 (10 points)

A manufacturer finds that the marginal revenue  $MR(x)$  in dollars for producing and selling  $x$  items *per* week is given by  $MR(x) = 20 + 0.02x$ . How much does the total revenue change if sales change from 1000 items *per* week to 2000 items *per* week?

Problem 6 (15 points)

The amount of material in a radioactive sample is decreasing at a rate proportional to the amount present. At time  $t = 1$  year there are 16 kilograms of the material. At time  $t = 5$  years there is just 1 kilogram of the material.

(a) How much material was there at time  $t = 0$  ?

(b) Find a formula giving the amount of material  $M(t)$  at any time  $t$ . (This should be a function which  $t$  can be “plugged into” to give a number which is the amount at that time. The constants in the formula should be given explicit numeric values.)

Problem 7 (10 points)

What is the area of the region in the plane between the graphs of  $f(x) = x^3 - 2x^2 + x + 1$  and  $g(x) = 2x^2 - 2x + 1$  ?



Problem 8 (12 points)

A ladder leans against the vertical side of a house. The bottom end of the ladder is on the horizontal ground. The ladder is 26 feet long. If the bottom end of the ladder is being pulled away from the house at a rate of 7 feet *per* minute, how fast is the upper end sliding down the side of the house at the moment when the upper end is 24 feet above the ground?

# SCRATCH PAPER