Mathematics 210, Summer 1999 (Wilson) Final Exam August 5, 1999

Your Name: _____

- 1. You have 75 minutes to work on this exam.
- 2. There are eight problems. Write your answers to them in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem. (The back of the last sheet is blank and can be used for scratch paper.)
- 3. On the back of this sheet is a table for the standardized normal random variable.
- 4. You may refer to notes you have brought in on up to three sheets of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score		
1	24			
2	24			
3	24			
4	24			
5	24			
6	23			
7	23			
8	24			
TOTAL	190			

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Problem 1 (24 points)

A Markov chain has the transition matrix $P = \begin{bmatrix} .6 & .4 \\ .3 & .7 \end{bmatrix}$.

(a) If this chain starts in state 1, what is the probability that it will be in state 2 after three transitions?

(b) Find the stable vector for this chain.

Problem 2 (24 points) A linear programming problem has constraints:

(a) On the axes below, plot <u>and shade in</u> the feasible set for this linear program. Be sure to (i) label the units on the axes, (ii) label the corners of the feasible set, and (iii) label each line as to which equation it comes from.



(b) Find the maximum value of y - x on the feasible set: Be sure to tell both what that maximum value is and also where it occurs. (If you find that there is no maximum, tell how you know that.)

(c) Find the minimum value of 2y + x on the feasible set: Be sure to tell both what that minimum value is and and also where it occurs. (If you find that there is no minimum, tell how you know that.)

Problem 3 (24 points)

A machine makes widgets. Each widget it makes has probability 0.95 of working properly. A lot of 10,000 widgets from this machine is tested. If X is the number of the tested widgets which work properly, what is $Pr[9500 \le X \le 9530]$?

Problem 4 (24 points)

Each day that you skip a class, there is a chance that the professor will give a "pop" quiz. If the probability of missing a quiz is 0.1 each time you skip class, and you skip 12 classes during the semester:

(a) What is the probability that you will miss exactly two quizzes?

(b) What is the expected number of quizzes you will miss?

Problem 5 (24 points)

There are three apples and two peaches in a box. You reach into the box and randomly pick up two pieces of fruit.

(a) What is the probability that both of the pieces you get are apples?

(b) What is the probability that the two pieces of fruit that you get are the same kind? (I.e, that either both are apples or both are peaches.)

Problem 6 (23 points)

Planning for retirement, you purchase an annuity. You agree to pay \$300 monthly for 30 years. The interest rate on the annuity is 6% per year. How much will the annuity pay?

Problem 7 (23 points)

You shop at three different CD stores. When you shop for CDs, 40% of the time you go to store A, 35% of the time to store B, and 25% of the time to store C. When you shop in store A, there is a 20% chance that you will buy a CD (and 80% that you will not.) At store B, the probability that you will buy a CD is 0.40. If you are shopping in store C the probability is 0.60 that you will buy a CD.

If you go shopping for a CD and buy one, what is the probability that you got it at store C?

Problem 8 (24 points)

You want to buy a house, and expect to finance it with a mortgage payable in monthly payments for 20 years, with an 8% annual interest rate. If you can afford a monthly payment up to \$800, what is the most expensive house you can buy? (Ignore down payment!)