Mathematics 210, Summer 1999 (Wilson)

Exam 2 July 22, 1999

Your Name: _____

1. There are six problems. Write your answers to them in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
2. On the back of this sheet is a table for the standardized normal random variable.
3. You have 75 minutes to work on this exam.
4. You may refer to notes you have brought in on one or two sheets of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	16	
2	18	
3	14	
4	17	
5	18	
6	17	
TOTAL	100	

Areas under the Standard Normal Curve, from $Z = 0$ to the given value of Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Problem 1 (16 points)
Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 5 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}.$$

(a) Find A^{-1}.

(b) Use A^{-1} to find the solutions to $Ax = b$, where

$$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Problem 2 (18 points)

A coin with probability 0.6 of coming up heads is flipped 1000 times. The random variable X is the number of heads observed.

(a) What are the expected value (mean) of X, the variance of X, and the standard deviation of X?

(b) Describe how one could compute exactly $Pr[580 \leq X \leq 610]$: Tell what formulas are needed, how to use them, and how to put the results together. You should provide a sufficiently detailed "recipe" that anyone who could use a calculator with buttons for addition, multiplication, raising to powers, and $C(n, r)$ could carry out the work.

(c) Use the normal approximation to the binomial random variable to find $Pr[580 \leq X \leq 610]$ to three decimal places.

Problem 3 (14 points)

The augmented matrix for a system of 3 equations in 4 unknowns has already been partially transformed toward reduced form, producing

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 7 & 3 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 2 & 6 & 4 \end{array}\right].$$

(a) Finish reducing this matrix.

(b) What are the solutions to the equations represented by this matrix? (Describe values for four unknowns x_1, x_2, x_3, and x_4.)

Note: This system does have solutions: If you find otherwise, check your work on part (a).

Problem 4 (17 points)

A random variable X can take on four different values: It takes the value 2 with probability 0.3, the value 4 with probability 0.2, the value 6 with probability 0.4, and the value 8 with probability 0.1. Find the expected value $E(X)$, the variance $V(X)$, and the standard deviation $\sigma(X)$. Be sure to label your answers.

Problem 5 (18 points)

A true/false test has 10 questions. You don't know the material too well, so you have to guess at the answers, but you do know enough about it that you have an 80% chance of guessing each one correctly.

(a) What is the probability that you get at least 9 questions right? (You can compute this exactly: Do not use a normal approximation.)

(b) You've heard some advance word on the grades, and know that you got at least 8 right: Now what is the probability that you got at least 9 right?

Problem 6 (17 points)

Your food cart offers two types of blended fruit "smoothies":

The Great Guzzle uses 9 ounces of bananas, while the Petite Perfection uses 5 ounces of bananas. The Great Guzzle uses 5 ounces of raspberries, and the Petite Perfection uses 8 ounces of raspberries. Each kind uses 6 ounces of strawberries.

You have available for each day's production 2250 ounces of bananas and 1800 ounces of strawberries and 2000 ounces of raspberries.

On the axes below, show graphically the allowable numbers of both kinds of smoothies that you can make in a day. (Be sure to label the axes: Tell which axis represents which quantity, and what the units are along each axis.)