Problem 1:
(a) We write a 3×6 matrix with A in the first three columns and the identity (3×3) in the last three:

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Subtract 5 times the first row from the second, and 3 times the first row from the third, to get

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -4 & -5 & 1 & 0 \\ 0 & 4 & -3 & -3 & 0 & 1 \end{bmatrix}.$$

Now subtract the third row from the second to get

$$\begin{bmatrix} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & -1 \\ 0 & 4 & -3 & -3 & 0 & 1 \end{bmatrix}.$$

Next we need to clear out the second column: Add the second row to the first, and subtract 4 times the second row from the third, getting

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & -1 & -2 & 1 & -1 \\ 0 & 0 & 1 & 5 & -4 & 5 \end{bmatrix}.$$

That gives us our leading 1 in the third row, so we are set to clear out the third column: Adding the third row to the second gives

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 3 & -3 & 4 \\ 0 & 0 & 1 & 5 & -4 & 5 \end{bmatrix}.$$

So now the inverse of A must be the last three columns of that matrix,

$$A^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 4 \\ 5 & -4 & 5 \end{bmatrix}.$$

(You can check that this is correct by computing $A\,A^{-1}$ which gives the 3×3 identity matrix.)

(b) We just multiply $A^{-1}b$, getting

$$\begin{bmatrix} -2 \\ 9 \\ 12 \end{bmatrix}.$$

In terms of the variables, $x_1 = 2$, $x_2 = 8$, and $x_3 = 5$.

Problem 2:
(a) This is a binomial random variable, with $n = 1000$ and $p = 0.6$. The expected value is $E(X) = n\,p = 1000 \times 0.6 = 600$. The variance is $V(X) = n\,p\,(1-p) = 600 \times 0.4 = 240$. The standard deviation is $\sigma = \sqrt{V(X)} = \sqrt{240} \approx 15.4919$.
(b) Since the variable is binomial, we can use the formula for a Bernoulli process to calculate the probability that it takes on any particular value: $Pr[X = k] = C(n,k)\,(0.6)^k\,(0.4)^{1000-k}$ for any value of k from 0 to 1000. The problem asks for $Pr[580 \leq X \leq 610]$. That is the sum of the probabilities $Pr[X = 580] + Pr[X = 581] + Pr[X = 582] + Pr[X = 583] + Pr[X = 584] + \ldots Pr[X = 610]$. So the answer can be found (in principle, but requiring a great deal of calculation!) by using the first formula for all of the values of k from 580 to 610 and then adding up the results.
(c) We approximate this binomial random variable by a normal random variable with the same mean and standard deviation. From (a) we know those are $\mu = 600$ and $\sigma = 15.4919$. The probability that X takes on a value between 580 and 610 (inclusive) is approximately the probability that a normal random variable with that mean and standard deviation takes on a value between 579.5 and 610.5. (I said in class that I would not take off points if you did not use the ± 0.5 correction!) That is the same as the probability that a standard normal random variable takes on a value between $\frac{579.5-600}{15.4919} = -1.3233$ and $\frac{610.5-600}{15.4919} = 0.6777$. We divide that into the probability of being between -1.3233 and zero, which is the same as the probability of being between zero and 1.3233 by the symmetry of the normal curve, and the probability of being between zero and 0.6777. Each of those can be read from the table, 0.4066 and 0.2517 are close enough, and we add them to get 0.6583 as the answer.

Problem 3:
(a) The matrix as given has leading entries in the first and second rows already in place, and the first column has been cleared out. We need to clear out the third column, where the leading entry for row two is. Subtract the second row from the first and subtract twice the second row from the third to get

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 4 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

That is in reduced form.
(b) We see that the second and fourth columns do not contain leading entries, so assign arbitrary values to the second and fourth unknowns x_2 and x_4. The first row of the reduced matrix corresponds to the equation $x_1 - 2x_2 + 4x_4 = 1$, so let $x_1 = 1 + 2x_2 - 4x_4$. The second row corresponds to the equation $x_3 + 3x_4 = 2$, so let $x_3 = 2 - 3x_4$.

Problem 4: I'll arrange the computations in columns. First, for the expected value,

x_i	p_i	$x_i \times p_i$
2	0.3	0.6
4	0.2	0.8
6	0.4	2.4
8	0.1	0.8

and we add the numbers in the last column to get $E(X) = 4.6$. Now, for the variance, we add more columns

$x_i - 4.6$	$(x_i - 4.6)^2$	$p_i(x_i - 4.6)^2$
-2.6	6.76	2.028
-0.6	0.36	0.072
1.4	1.96	0.784
3.4	11.56	1.156

Adding up the numbers in the last column gives us $V(X) = 4.04$ for the variance, and taking the square root of that gives the standard deviation $\sigma \approx 2.0100$.

Problem 5: This is a binomial random variable with $n = 10$ and $p = 0.8$.
(a) The probability of getting at least 9 questions right is the probability of getting 9 right plus the probability of getting 10 right: $C(10,9)(0.8)^9(0.2)^1 + C(10,10)(0.8)^1 0(0.2)^0 = 0.2684 + 0.1074 = .3758$.
(b) The probability of getting 9 or more right *given* that at least 8 are right is

$$\frac{P[(\text{at least } 9) \cap (\text{at least } 8)]}{P[\text{at least } 8]}.$$

If you get 9 or more right then you certainly get 8 or more right, so the event in the numerator is the same as the event in part (a). We compute the denominator as follows: Getting at least 8 right is the same as getting exactly 8 right or getting 9 or more right. The probability of getting exactly 8 right is $C(10,8)(0.8)^8(0.2)^2 = 0.3020$, and the probability of getting 9 or more right is what we had in (a). So the answer is

$$\frac{0.3758}{0.3020 + 0.3758} = 0.5544$$

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Problem 6: I'll use G for the number of Great Guzzle smoothies you make in a day, and P for the number of Petite Perfections. From the amount of bananas available we get that $9G + 5P \leq 2250$. From the strawberry amount we get $6G + 6P \leq 1800$. From the raspberry restriction we get $5G + 8P \leq 2000$. It is also implicit that we don't use negative amounts of the ingredients, i.e. $G \geq 0$ and $P \geq 0$. We plot the lines that come from changing those inequalities to equations and then see which side of each line is included. I have indicated the resulting region using the cross-hatched lines below.