Problem 1:

(a) There are $C(36,3)$ ways to choose the three cards. They could all be black in $C(18,3)$ ways or they could all be red in $C(18,3)$ ways. The probability that all three are the same color is $\frac{C(18,3)+C(18,3)}{C(36,3)}$ which works out to about 0.2286.

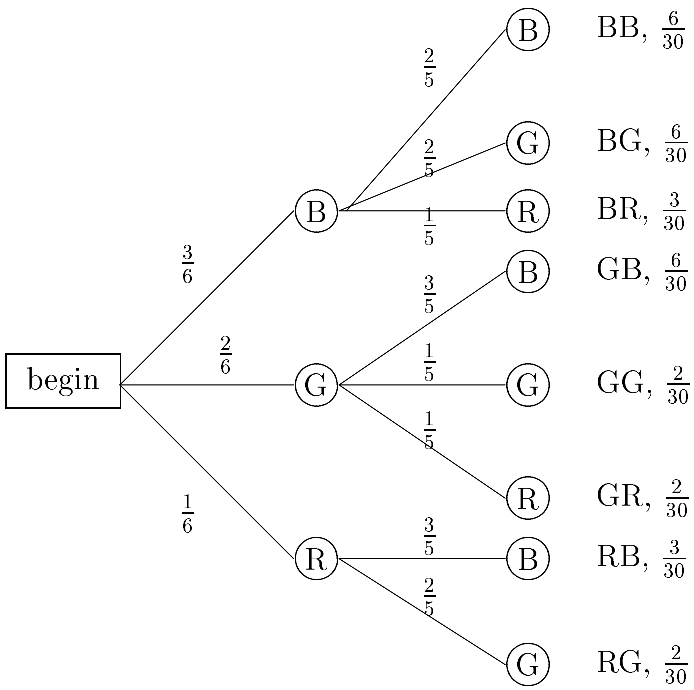
(b) There are several ways to do this. One can think of a tree for this stochastic process (but not draw it all, as it is large!) or one can compute using the formula for conditional probability. But the easiest is this: After the first card has been dealt, and it is an eight, there are 35 cards left and 4 of them are nines. So the probability is just $\frac{4}{35}$.

(c) The combinations which add up to give 8 are 2 and 6 (in either order), 3 and 5 (in either order), and 4 and 4. If we think of dealing one card and then the other, to get a 2 and a 6 in that order we have four ways of getting the 2 on the first card and four ways of getting the 6 on the second card: Thus there are 16 ways to get the 2 on the first card and the 6 on the second. The same reasoning applies to getting the (ordered) combinations 3 and 5, 5 and 3, and 6 and 2, for a total of $16+16+16+16=64$ ways to get all of those combinations. The combination 4 and 4 is different, though: If we get a 4 on the first card, there are only three 4's left. So there are only 12 ways to get the combination 4 and 4. Hence there are $64+12=76$ ways to get a combination whose sum is 8. Now how many ways are there to deal the two cards? We can use the formula $P(36,2)$ or just think it out: There are 36 choices for the first card, and 35 for the second, so there are $36\times 35=1260$ ways to choose the two cards. Now we can compute the probability, $\frac{76}{1260}\approx 0.0603$.

(There were many, very different, answers to this problem. One way to check: Apply the same logic to all the other possible sums, from $2+2=4$ to $10+10=20$, and add the results. Using this model the sum is 1.)

Problem 2:

(a)



(b) $Pr[\text{second is green} \cap \text{first is red}] = Pr[\{RG\}] = \frac{2}{30}$. $Pr[\text{first is red}] = \frac{3}{30} + \frac{2}{30} = \frac{5}{30}$. Thus $Pr[\text{second is green}|\text{first is red}] = \frac{2/30}{5/30} = \frac{2}{5}$.

(c) We can use the same numerator as in (b) but need to divide by $Pr[\text{second is green}]$ rather than $Pr[\text{first is red}]$. $Pr[\text{second is green}] = \frac{6+2+2}{30} = \frac{10}{30}$, so the answer is $\frac{2/30}{10/30} = \frac{2}{10}$.

Problem 3:

This is a pretty direct application of Bayes' Formula. If we let A be the event that an item was produced on assembly line A, and similarly for the other lines, and X be the event that it is defective, we are given: $Pr[A] = .40$, and $Pr[X|A] = .05$; $Pr[B] = .20$, and $Pr[X|B] = .10$; $Pr[C] = .30$, and $Pr[X|C] = .15$; and finally $Pr[D] = .10$, and $Pr[X|D] = .25$. What we are asked for is $Pr[C|X]$, which we can calculate as

$$Pr[C|X] = \frac{.15 \times .30}{.05 \times .40 + .10 \times .20 + .15 \times .30 + .25 \times .10} = \frac{9}{22}.$$

Problem 4:

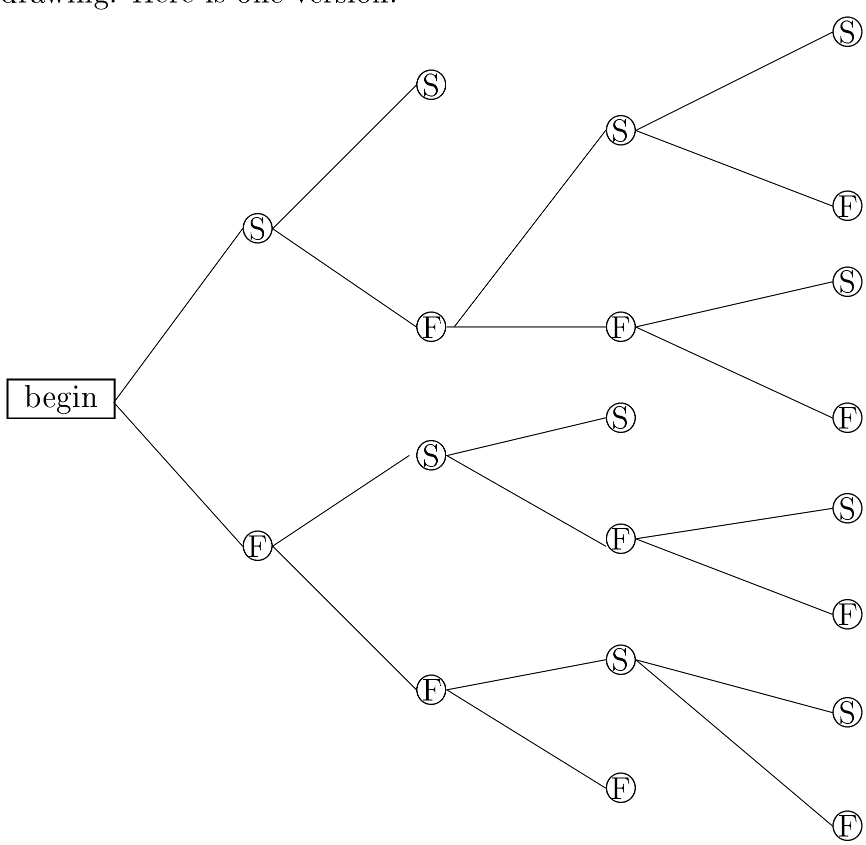
(a) If we let x be the probability for the outcome 1, then the probability of 2 is $2x$, the probability of 3 is $3x$, and so on through the probability of 10 which is $10x$. Their sum must be 1, so we have the equation $x + 2x + 3x + \ldots + 9x + 10x = 1$ which simplifies to $55x = 1$. Thus $x = \frac{1}{55}$. Now we can write the probabilities for each of the outcomes: I'll simplify to "1" for the event "$\{1\}$", etc.: $Pr[1] = \frac{1}{55}$, $Pr[2] = \frac{2}{55}$, $Pr[3] = \frac{3}{55}$, $Pr[4] = \frac{4}{55}$, $Pr[5] = \frac{5}{55}$, $Pr[6] = \frac{6}{55}$, $Pr[7] = \frac{7}{55}$, $Pr[8] = \frac{8}{55}$, $Pr[9] = \frac{9}{55}$, and $Pr[10] = \frac{10}{55}$.

(You should be careful what you write! A lot of students wrote things like $3 = \frac{3}{55}$, which is clearly false...)

(b) The probability that the number is even is just the sum of the probabilities above going with the even outcomes: $\frac{2+4+6+8+10}{55} = \frac{30}{55} = \frac{6}{11}$.

Problem 5:

(a) I think it would be hard to be sure just where sequences of calls come to a stop without a drawing. Here is one version:

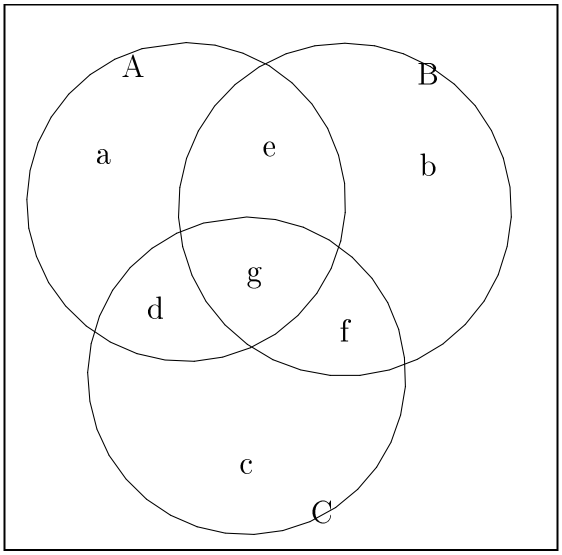


We can count 11 "endings" in the diagram.

(b) Looking at those 11 we find that 7 of them include two or more sales.

(c) Only two of the paths stop after exactly three calls.

Problem 6:
We are asked whether A and C are independent. The natural way to check that is to compute $Pr[A \cap C]$ and compare it to $Pr[A] \times Pr[C]$: If they are equal, the events are independent, and if they are not equal, the events are not independent. We draw and label a Venn diagram:



The probability we need to find, $Pr[A \cap C]$, corresponds to $d+g$ in the picture. Now we can use the facts given in the problem to write equations $a+e+d+g = .45$, $b+e+g+f = .50$, $c+d+g+f = .30$, $e+g = .15$, $f+g = .10$, and $g = .05$. From the last three equations we get $f = .05$ and $e = .10$, which we can use to simplify the first three to $a+d = .30$, $b = .30$, and $c+d = .20$. The last of the facts in the problem tells us $a+b+c+d+e+f+g = .90$. Putting in the other numbers we have so far, that simplifies to $a+c+d = .90-.30-.10-.05-.05 = .40$. If we add the equations $a+d = .30$ and $c+d = .20$ we get $a+c+2d = .50$, and if we subtract the previous result we get $d = .10$. That makes $d+g = .10+.05 = .15$, so $Pr[A \cap C] = .15$. But $Pr[A] \times Pr[C] = .45 \times .30 = .135$ which is not the same as .15, so we conclude the events are <u>not</u> independent.
(Here is a different way to do Problem 6: If we assume that A and C <u>are</u> independent, we get $Pr[A \cap C] = .135$. Using that we can fill in the picture above, starting in the center and working out. We get $g = .05$, $f = .05$, $e = .10$, $d = .0835$ (from the fact that $d+g = .135$), etc. When you get through, if you add $a+b+c+\ldots+g$, you don't get .90. So the assumption is incompatible with the given data.)