AnswersExam II 10/18/96Mathematics 210 (Wilson)Problem 1: There are 3 red, 3 blue, 2 white balls and we draw two of them.

(a) The probability both are blue:

The total number of ways to draw two balls out of 8 is  $C(8,2) = \frac{8!}{6! \times 2!} = 28$ . The number of ways to draw both of them from the three blue ones is  $C(3,2) = \frac{3!}{1! \times 2!} = 3$ . Thus the probability that both are blue is  $\frac{3}{28} \approx 0.1072$ .

(b) The probability both are blue, given neither is red:

The probability that neither is red is  $\frac{C(5,2)}{C(8,2)} = \frac{10}{28} \approx 0.3571$ . The probability that both are blue AND neither is red (which is the same as both are blue since that implies neither is red) is the answer from (a),  $\frac{3}{28}$ . Thus  $Pr[\text{both are blue}|\text{neither is red}] = \frac{3/28}{10/28} = \frac{3}{10} = 0.3$ . (c) The probability that both are blue, given that at least one is blue:

The probability that at least one is blue is the probability that one (exactly) is blue plus the probability that both are blue. The probability that exactly one is blue is  $\frac{C(3,1)C(5,1)}{C(8,2)} = \frac{15}{28} \approx 0.5357$ . The probability that both are blue was found in (a). Thus the probability that at least one is blue is  $\frac{3}{28} + \frac{15}{28} = \frac{18}{28} \approx 0.6429$ . The probability that both are blue AND at least one is blue is the same as the probability that both are blue, found in (a). Then  $Pr[\text{both are blue}|\text{at least one is blue}] = \frac{3/28}{18/28} = \frac{3}{18} = \frac{1}{6} \approx 0.1667$ .

Problem 2:

(a) Letting RC mean Red Chocolate, GC mean Green Chocolate, and RP mean Red Peanut-Butter, with 4 of the first and three of each\_of the others and we pick two:



(b)  $Pr[\text{both red}] = Pr[RC, RC] + Pr[RC, RP] + Pr[RP, RC] + Pr[RP, RP] = \frac{12}{90} + \frac{12}{90} + \frac{12}{90} + \frac{12}{90} + \frac{6}{90} = \frac{42}{90} \approx 0.4667.$ 

Problem 3: Questions from file A have 20% easy, 50% medium, and 30% hard; questions from file B have 40% easy, 40% medium, and 20% hard; I pick from file A 40%of the time.

(a) Pr[easy] = Pr[easy from A]Pr[easy from A] + Pr[easy from B]Pr[easy from B] $= (0.2 \times 0.4) + (0.4 \times 0.6) = 0.08 + 0.24 = 0.32.$ (b)

$$Pr[\mathbf{from } \mathbf{B}|\mathbf{easy}] = \frac{Pr[\mathbf{easy}|\mathbf{from } \mathbf{B}]Pr[\mathbf{from } \mathbf{B}]}{Pr[\mathbf{easy}|\mathbf{from } \mathbf{B}]Pr[\mathbf{from } \mathbf{B}] + Pr[\mathbf{easy}|\mathbf{from } \mathbf{A}]Pr[\mathbf{from } \mathbf{A}]}$$
$$= \frac{0.4 \times 0.6}{(0.4 \times 0.6) + (0.2 \times 0.4)} = \frac{0.24}{0.32} = 0.75$$

Problem 4: Each ticket has probability 0.1 of winning. Buying n tickets, the number of winning tickets is a binomial random variable with probability 0.1 of success. The probability that at least one is a winner is most easily computed by subtracting the probability that all are losers from 1,  $1 - (0.9)^n$ . Thus we have to find the smallest n such that  $1 - (0.9)^n \ge 0.75$ . Rearranging, that is  $(0.9)^n \le 0.25$ . Trying some values with a calculator, for n = 10 we find  $(0.9)^{10} \approx 0.349$  so n = 10 is not enough. For n = 15 we find  $(0.9)^{15} \approx 0.206$  so n = 15 may be too big. Trying values in between we find that n = 14is the smallest that works: At n = 13,  $(0.9)^{13} \approx 0.254$  just misses.

Problem 5: The coin has probability  $\frac{3}{5}$  of giving heads. Since we flip it until either we get tails or three heads in a row, the game can produce T or HT or HHT.

(a) The variable X is the number of flips so it takes on the values 1, 2, and 3. (b) T has probability  $\frac{2}{5}$  so that is Pr[X = 1]. HT has probability  $\frac{3}{5}\frac{2}{5} = \frac{6}{25}$  so that is Pr[X = 2]. X = 3 happens if the first two flips produce HH, which has probability  $\frac{3}{5}\frac{3}{5}\frac{3}{5}=\frac{9}{25}$ .

(c) It will make the calculation of expected value easier if we write all the probabilities with the same denominator:  $Pr[X = 1] = \frac{10}{25}$ ,  $Pr[X = 2] = \frac{6}{25}$ ,  $Pr[X = 3] = \frac{9}{25}$ . Then  $E[X] = 1 \times \frac{10}{25} + 2 \times \frac{6}{25} + 3 \times \frac{9}{25} = \frac{10+12+27}{25} = \frac{49}{25} = 1.96$ 

(d) Because of the way the numbers work out it is probably easier to compute the variance using the probabilities in decimal form: Pr[X = 1] = 0.4, Pr[X = 2] =0.24, Pr[X=3] = 0.36. Then the variance is  $0.4(1-1.96)^2 + 0.24(2-1.96)^2 + 0.36(3-1.96)^2$  $= 0.4(-0.96)^2 + 0.24(0.04)^2 + 0.36(1.04)^2 = 0.4 \times 0.9216 + 0.24 \times 0.0016 + 0.36 \times 1.0816 \approx$ 0.3686 + 0.0004 + 0.3894 = 0.7584.

(e) The standard deviation is the square root of the variance,  $\sqrt{0.7584} \approx 0.8709$ .

Problem 6: The given normal random variable has  $\mu = 10$  and  $\sigma = 3$ . We will have to convert in the form  $Z = \frac{X-\mu}{\sigma}$  before using the table. (a)  $Pr[10 \le X \le 16] = Pr[\frac{10-10}{3} \le Z \le \frac{16-10}{3}] = Pr[0 \le Z \le 2]$ . That will be exactly the

entry for 2.00 in the table, 0.4772.

(b)  $Pr[13 \le X \le 16] = Pr[\frac{13-10}{3} \le Z \le \frac{16-10}{3}] = Pr[1 \le Z \le 2]$ . We can find the area between 1 and 2 by subtracting the area between 0 and 1 from the area between 0 and 2. Using the table we get 0.4772 - 0.3413 = 0.1359.