Answers, Exam 1 for Wilson's fall 1996 Math 210

There are in general several methods for doing each problem, and certainly many ways to write out each method. You did not necessarily have to use my methods, or my form of writing, but you did have to have a method which really works.

Problem 1: Most people find it helpful to draw a picture for this kind of problem, but here is a version to show you *can* do it just with words.

(a) There are 131 = 200 - 69 students who own at least one of these devices. The union of the three sets (the TV owners T, the radio owners R, and the CD owners C) has  $n(T \cup R \cup C) = n(T) + n(R) + n(C) - n(T \cap R) - n(T \cap C) - n(R \cap C) + n(T \cap R \cap C)$  students in it. We know that is 131. Since no students own both radio and CD player,  $n(R \cap C)$  and  $n(T \cap R \cap C)$  are both zero. We also are given that n(t) = 98, n(R) = 32, and n(C) = 39. Thus  $131 = 98 + 32 + 39 - n(T \cap R) - n(T \cap C)$ . Rearranging,  $n(T \cap R) + n(T \cap C) = 38$ . Thus 38 students own a TV set and some other device, out of 98 who own a TV set, so 60 own just a TV set.

(b) If 21 own a CD player but not a TV set, out of 39 who own a CD player, then 18 must own both a CD player and a TV set. We already got that 38 own a TV set and something else, and none own all three devices, so 38 - 18 = 20 of them must own a TV set and a radio.

Problem 2:

(a) Here is a way to draw the diagram. I have used R, W, and G to denote drawing a Red, White, or Green ball respectively. The branches end wherever we have had two red or one green, and the choices at each stage are made from only the balls which are left after the ones drawn so far on that path.



(b)  $\{RR, RWR, RWG, RG, WRR, WRG, WG, G\}$ 

## Problem 3:

- (a)  $\{HH, HTHH, HTHT, HTT, THH, THTH, THTT, TT\}$
- (b) 8 (just by counting)
- (c) 1 (just by counting)
- (d) 0 (just by counting)

## Problem 4:

(a) Let  $w_i$  be the weight or probability assigned to side *i*. From the fact that we are assigning probabilities,  $w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1$ . The problem tells us that  $w_1 = w_2$ , and that  $w_3 = w_4 = w_5 = w_6$ , and that  $w_1 = 4w_3$ . Substituting the last two facts in the first equation gives  $4w_3 + 4w_3 + w_3 + w_3 + w_3 + w_3 = 1$ , or  $12w_3 = 1$ , so  $w_3 = \frac{1}{12}$ . Thus  $w_1$  and  $w_2$  must be  $\frac{4}{12} = \frac{1}{3}$  while each of the others is  $\frac{1}{12}$ .

(b) 
$$w_4 + w_5 + w_6 = \frac{3}{12} = \frac{1}{4}$$
.

# Problem 5:

(a) Order matters (since clearly if you and I swap keys the results are different) so the

number of outcomes is the number of ways of choosing 4 things from 4 where order matters,  $P(4,4) = \frac{4!}{0!} = 4! = 24.$ 

(b) The outcomes where I get my own keys back are the ways that happens <u>and</u> the other three keys are distributed in some way: There is only one way for the first part of that to happen, the second can happen P(3,3) = 3! = 6 ways, and because of the "and" we multiply, so there are  $1 \times 6 = 6$  such outcomes. (Thus the probability that I get my own keys, no matter what happens to the others, would be  $\frac{6}{24} = \frac{1}{4}$ , but you were not asked for that.)

(c) Think of this as a three step process: First pick which two key rings get returned to their owners, then return those two, then return the other two in such a way that they <u>don't</u> go to their owners. There are  $C(4,2) = \frac{4!}{2!2!} = 6$  ways to do the first step. The second, give two keys (already chosen) to their rightful owners, can only be done in one way. The third, give two keys to the remaining owners so that they <u>don't</u> match, can only be done way. Multiplying  $6 \times 1 \times 1$  gives 6 ways to do this.

#### Problem 6:

There are 15 balls, we draw 3, so there are  $C(15,3) = \frac{15!}{12!3!} = 455$  ways to select them.

(a) The event "all three green" occurs C(6,3) = 20 ways, so it has probability  $\frac{20}{455} \approx 0.044$ . (b) "At least one red" occurs if we have one red with two green <u>or</u> two red with one green <u>or</u> three red. You can compute how many ways those happen, but it is easier to note that at least one red is just the opposite of all three green, so the probability must be  $1 - \frac{20}{455} \approx 0.956$ .

## Problem 7:

We observe "which number came from which die" so the outcomes are pairs, a number from 1...6 from the red die and a number from 1...6 from the green die. There are 36 such pairs, and from what the problem says each is equally likely.

(a) The two numbers can be the same if they are both 1, or if they are both 2, ..., or if they are both 6. Thus there are 6 ways for this to happen, and the probability is  $\frac{6}{36} = \frac{1}{6}$ . (b) The outcomes for which the product is 19 or more are (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), and (6,6), eight in all. Thus the probability is  $\frac{8}{36} = \frac{2}{9}$ .

#### Problem 8:

(a) There are only four results as they are originally described: (i) all three folk, (ii) two folk and one rock, (iii) one folk and two rock, and (iv) all three rock.

(b) These results do not take into account the fact that a folk ticket is three times as likely to be picked as a rock ticket: For example, it treats "all three folk" and "all three rock" in the same way. Choosing three folk tickets (from 18) can be done in many more ways (C(18,3) = 816) than choosing three rock tickets (from 6, C(6,3) = 20 ways).

(c) The problem with the original description was that it just noted kinds of tickets and not tickets. If we describe the outcomes as triples of individual tickets, as if we had numbered the 18 folk tickets to tell them apart and had numbered the 6 rock tickets to tell them apart, we have C(24,3) = 2024 outcomes (instead of four) and they are equally likely.

(d) We get three tickets for the same concert either as three for the folk concert or three for the rock concert. The first occurs in C(18,3) ways and the second in C(6,3) ways, so there are C(18,3) + C(6,3) = 816 + 20 = 836 such outcomes and the probability is  $\frac{836}{2024} \approx 0.4032$ .

(e) We have already calculated the number of ways of getting three rock tickets as C(6,3) = 20, so the probability is  $\frac{20}{2024} \approx 0.0099$ .