Mathematics 210, Lecture 3 (Wilson)

Final Exam 12/16/96

Your Name: ______________________________

1. There are ten problems. Write your answers to them in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
2. On the back of this sheet is a table for the standardized normal random variable.
3. You may use a calculator, but if you do so be sure to show (a) what you asked it to do, (b) what it gave as a response, and (c) any interpretation you had to make to answer the problem based on that response.
4. You have two hours to work on this exam.
5. You may refer to notes you have brought in on up to four sheets of paper, as announced in class.

BE SURE TO SHOW YOUR WORK:
A NUMERICALLY CORRECT ANSWER MAY RECEIVE REDUCED OR ZERO CREDIT IF IT IS NOT SHOWN TO BE BASED ON CORRECT REASONING!

Problem	Points	Score
1	12	
2	18	
3	15	
4	15	
5	16	
6	15	
7	14	
8	15	
9	16	
10	14	
TOTAL	150	

Areas under the Standard Normal Curve, from $Z = 0$ to the given value of Z

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Problem 1 (12 points)

A normal random variable X has expected value $\mu = 15$ and variance $V = 4$.

(a) What is $Pr[13 \leq X \leq 20]$?

(b) What is $Pr[X \geq 17]$?

Problem 2 (18 points)

A Markov chain has transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{bmatrix}$$

(a) This is an absorbing Markov chain: How can you tell that from P?

(b) P is not in canonical form: Change it into canonical form.

(c) Find the fundamental matrix for the canonical form you gave in (b).

(d) If the chain starts in state 3, what is the expected number of times it will be in state 1 before it reaches an absorbing state?

(e) If the chain starts in state 3, what is the expected number of transitions before it first reaches an absorbing state?

Problem 3 (15 points)

Let

$$A = \begin{bmatrix} 0 & 2 & 7 \\ 1 & -1 & 1 \\ 0 & 1 & 4 \end{bmatrix}.$$

(a) Find A^{-1}.

(b) Solve the system of equations

$$\begin{aligned} 2y + 7z &= 1 \\ x - y + z &= 2 \\ y + 4z &= 3 \end{aligned}$$

(c) Solve the system of equations

$$\begin{aligned} x + y - z &= 3 \\ x - y - 3z &= -1 \\ 2y + 2z &= 4 \end{aligned}$$

Problem 4 (15 points)

You buy a house which costs $150,000. You finance it with a mortgage for 20 years, paid and compounded monthly, with an annual interest rate of 9%.

(a) What will be the monthly mortgage payment?

(b) What is the total of your monthly payments? How much of your total went for interest? (The total will be made up of the interest and the $150,000 cost of the house.)

Problem 5 (16 points)

An urn contains 3 red and 4 white balls. As an experiment we remove three balls from the urn, one after another, chosen randomly and without replacement, and note the colors. We write an outcome from the experiment as a triple of letters: "WRW" for example would mean that we drew first a white ball, second a red ball, and third another white ball.

(a) List the members of the sample space for this experiment.

(b) Draw a tree diagram for this experiment.

(c) What is the probability of the outcome "WRW"?

(d) What is the probability that two white balls will be chosen (without regard to where in the sequence of three they occur)?

(e) What is the probability that the first ball chosen was white, given that two white balls were chosen?

Problem 6 (15 points)

An unfair coin has Pr[heads]= $\frac{1}{4}$ and Pr[tails]= $\frac{3}{4}$. We flip the coin 6 times.

(a) What is the probability that the coin shows heads *exactly* 5 times?

(b) What is the probability that the coin shows heads *at most* four times?

(c) What is the expected number of times that the coin shows heads?

Problem 7 (14 points)

A cubical die has the numbers 1, 1, 1, 2, 2, and 3 on its six faces. The six faces are equally likely to come up when the die is rolled. We roll the die, and note as a random variable X the number on the top face.

(a) What are the values X can take on?

(b) Give the probability density function for X.

(c) What is the expected value of X?

(d) What is the variance of X?

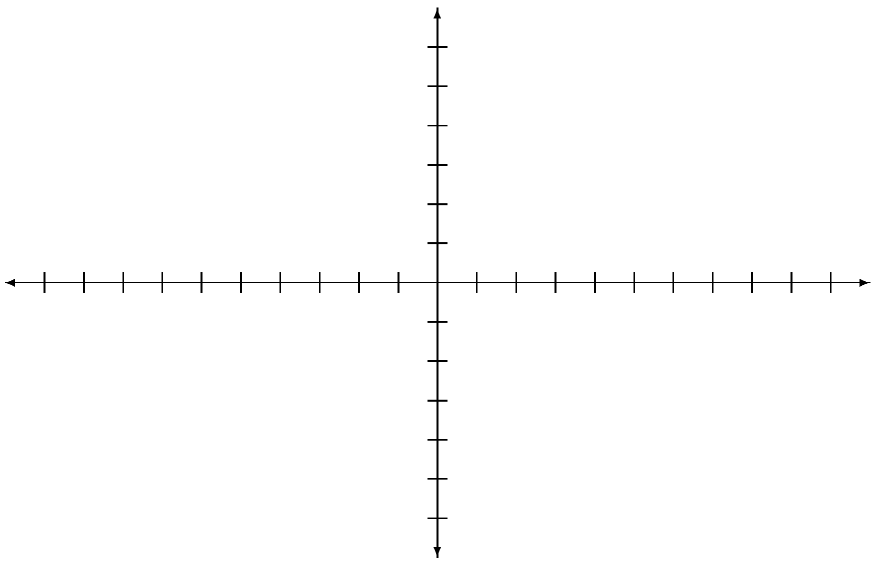
(e) What is the standard deviation of X?

Problem 8 (15 points)

The feasible set S for a linear programming is defined by

$$\begin{aligned} -2x - y - 4 &\leq 0 \\ 3x - y - 9 &\leq 0 \\ x - y - 1 &\leq 0 \end{aligned}$$

(a) Graph S on the axes provided. Be sure to indicate the units along the axes.



(b) Find the corner points of S: Label each corner on your graph above, and give here the (x, y) coordinates for each. Be sure to indicate which coordinates go with which corners!

(c) Find the minimum value of $y - 2x$ on S or show it does not exist. If it does exist be sure to show how you know what that minimum is and where it occurs.

(d) Find the maximum value of $y - 2x$ on S or show it does not exist. If it does exist be sure to show how you know what that maximum is and where it occurs.

Problem 9 (16 points)

A Markov chain has as its transition matrix

$$P = \begin{bmatrix} .5 & .5 \\ .75 & .25 \end{bmatrix}$$

(a) Is this Markov chain regular? How can you tell?

(b) Find the vector of stable probabilities for this Markov chain. (If you use your calculator be sure to tell exactly how you use it and how you interpret what it says.)

Problem 10 (14 points)

You make a deposit in a savings account. You deposit \$1000 on January 1, 1997. The interest rate is 6% on an annual basis, compounded quarterly.

(a) What is the annual percentage yield for this account?

(b) How much money will there be in the account on January 1, 1999?

(c) What is the present value on January 1, 1997, of the amount which will be in the account on January 1, 1999? (Use the same interest rates and time intervals as in (b).)