Mathematics 210, Lecture 3 (Wilson)

Exam IV 12/6/96

Your Name: ______________________________

1. There are FIVE problems. Write your answers to them in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
2. If you need scratch paper, please ask for it.
3. You have 50 minutes to work on this exam.
4. You may refer to notes you have brought in on one sheet of paper, as announced in class.

BE SURE TO SHOW YOUR WORK: YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	12	
2	18	
3	16	
4	18	
5	16	
TOTAL	80	

Problem 1 (12 points)
You plan to buy a vacation home in 5 years, which you estimate will cost \$120,000. In order to accumulate the money you plan to make deposits in a savings account for five years, each deposit the same amount. You will make payments every three months, starting January 1, into an account which collects interest at an annual rate of 8% compounded quarterly. How much must each deposit be in order to reach your goal?

Problem 2 (18 points)

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

is the transition matrix for a Markov chain. Find the vector of stable probabilities for this Markov chain.

Problem 3 (16 points)
A couple bought a house for \$70,000 in 1988 and sold it for \$108,000 in 1995.

(a) What was the effective annual rate of interest on this real estate investment? (I.e. what annual rate, if they had invested the \$70,000 in 1988, would have yielded \$108,000 in 1995?)

(b) If, instead of buying the house, they had in 1988 invested some amount of money at 5% annual rate, compounded quarterly, how much would they have had to invest to produce the same \$108,000 in 1995?

Problem 4 (18 points)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

is the transition matrix for a Markov chain.
Suppose the chain starts in state 2:

(a) What is the expected number of times the chain will be in state 2 before it reaches an absorbing state?

(b) What is the expected number of times the chain will be in state 3 before it reaches an absorbing state?

Problem 5 (16 points)
You purchase an annuity for retirement money, paying \$100 per week for 40 years with an annual interest rate of 5.2%.

(a) What will the annuity be worth when all payments are made?

(b) What is the present value of the annuity (at the time you start buying it)?