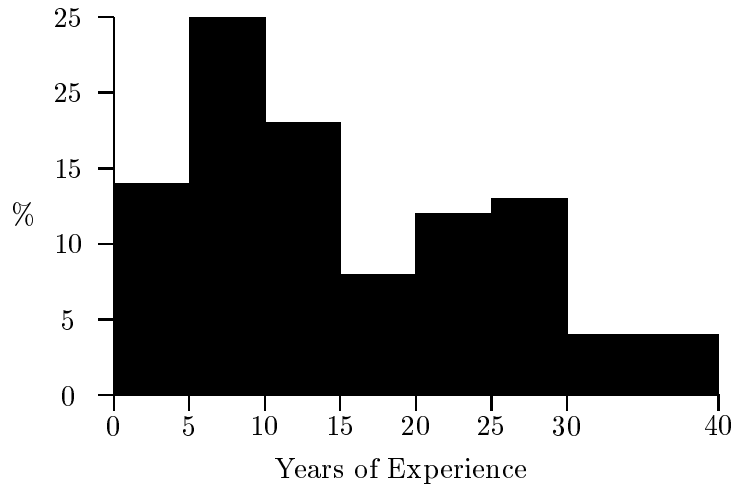
Mathematics 132 (Wilson) **Your Name:** _____

Final Exam
Summer, 1997

- This exam is a "take-home" exam. You are allowed to use any class notes you may have taken, your homework, and any class handouts for reference, but no other books or materials.
- If you have questions about the problems, either having to do with interpretation or how to answer them, please ask me: I will be available during regular class time and office hours from now until when the exam is due. You may also contact me by email or telephone, either with a question or to arrange to meet. My email address is wilson@math.wisc.edu, my Van Vleck office (411) telephone is 263-5944, and my home telephone is 274-3746.
- If you have a "trivial" question with regard to something like a detail of Excel you may ask others: I am relying on you to use this to get quick answers to simple things you could have read in the on-screen help or in the class handouts, and not to collaborate on how to do the problems.
- Please write your answers legibly and in understandable English. If something is ambiguous you can't count on my interpreting it the way you meant it.
- For answers where you use Excel, don't forget to (a) print all of your spreadsheets with gridlines showing and (b) give me printouts of both the results and the formulas.
- There are six problems: Be sure you don't miss seeing one on the back of a sheet!
- **Your answers including all computer printouts are due at the normal end-of-class time (9:55 AM) on Thursday, August 7, 1997.**

Problem 1:

Recent data on how long teachers in the Madison school system have been teaching are closely represented by the histogram shown. There were 2,287 teachers represented in these data.



(a) Estimate the percentage of Madison teachers who have up to 15 years of experience.

(b) Which group contains more teachers, or do they have the same number:

(i) Those with 10-15 years of teaching experience

(ii) Those with 15-25 years of teaching experience

(c) Which group contains more teachers, or do they have the same number:

(i) Those with 15-20 years of teaching experience

(ii) Those with 30-40 years of teaching experience

(d) About how many teachers have 25 or more years of experience? (Give a number of teachers, not a percentage.)

(e) Estimate the mean and median for the years of experience among Madison teachers. For each give the number of years, mark it on the histogram (turn in either this sheet or a copy of it), and tell how you got your answer and how accurate you think it might be.

Problem 2:

For each of the following different orderings, either give an example of a set of five numbers (some of which might be the same) such that the mean, median, and mode of the numbers are in the given order, or tell why that cannot be done:

$$\begin{aligned}
\text{mean} &< \text{median} < \text{mode} \\
\text{mean} &< \text{mode} < \text{median} \\
\text{median} &< \text{mean} < \text{mode} \\
\text{median} &< \text{mode} < \text{mean} \\
\text{mode} &< \text{mean} < \text{median} \\
\text{mode} &< \text{median} < \text{mean}
\end{aligned}$$

Problem 3:

> You may use either a spreadsheet or algebraic calculations on this problem: If you use algebra be sure to show your work and explain your reasoning; If you use a spreadsheet be sure to include the spreadsheet with your answers, showing both the results and the formulas. If you use a spreadsheet be sure to make the amounts initially invested as well as the interest rates easily changed.

You can invest in either of two accounts.
In account A you deposit an initial amount to begin with and then, at the end of each month, 1% of the amount which was in the account at the beginning of the month will be added to what you have in the account.
In account B you deposit an initial amount to begin with and then, at the end of each month, 3% of your original amount is added to what you have in the account.

(a) If you have \$1000 to invest, which account should you choose in order to have the most money in the account at the end of 12 months? How much money would each account have at that time if you had started each with \$1000?

(b) If you have \$1000 to invest, which account should you choose in order to have the most money in the account at the end of 240 months? How much money would each account have at that time if you had started each with \$1000?

(c) Which account should you choose if you are investing for retirement, if you do not expect to retire for 35 years?

(d) If you put \$1000 in each account, so that they start with the same amount, <u>when</u> will they again have the same amount? <u>How much</u> money will be in each account at that time?

Problem 4:

A quantity is given as a function $f(t)$ of time, where $f(t)$ has the effect that in each unit of time $k\,f(t)$ is added to $f(t)$ for some fixed constant k. (You might think of this as representing money in an account with compound interest, or the amount of some radioactive material, or the population of some species.) For what values of k will the graph of $f(t)$ be concave up? For what values of k will the graph of $f(t)$ be concave down? Are there any values of k which for which the graph is neither concave up nor down?

Problem 5:

Consider two classes, one a math class and the other a history class. Half of the students in the math class are also in the history class. Three sevenths of the students in the history class are also in the math class. If M represents the number of students in the math class:

(a) How many students are there in the history class? (This will depend on M.)

(b) How many students are there in the two classes together?

(c) Can there be exactly 26 students in the math class? Why or why not?

(d) Can there be exactly 49 students in the history class? Why or why not?

Problem 6:

Species living in an area interact so that the population of one affects the population of another. (You might think of this as being like a field with foxes and rabbits: The more rabbits there are, the better fed the foxes are and hence the fox population grows. The more foxes there are, however, the more rabbits become fox food and the rabbit population declines. Clearly this ignores many other factors which affect the populations but it can provide some insight nonetheless.) Suppose there are two interacting species, call them species A and species B. Suppose that we record the population of each species once a month and we find that the populations at the end of the month depend on the populations at the beginning of the month in this way:

> The population of species A at the end of a month is 0.4 times the population of A at the beginning of the month plus 0.2 times the population of B at the beginning of the month plus 300.
>
> The population of species B at the end of a month is 0.5 times the population of B at the beginning of the month plus 0.6 times the population of A at the beginning of the month minus 200.

(It would appear these species are symbiotic, in that more of either one increases the number of the other. For foxes and rabbits one of the multipliers would be negative to model the impact of foxes on rabbits.)

(a) Set up a spreadsheet which gives the population of each species month-by-month for 24 months, starting with a population of 3000 for species A and a population of 2000 for species B. Make sure that all of the numbers in the population formulas as well as the beginning populations are easily changed. Include a chart showing both the populations: Choose a form for the chart which shows how they populations compare at any time, and lets you see when one is larger than the other.

(b) For the data given, what are the populations at the end of 24 months? What was the highest population for each species, and when did it reach that peak? What do you think will happen to these species in the future?

(c) In the rule for the population of species B above, change the minus 200 to plus 200. What happens to your results? What do you think would happen to the species in the long run with these data?

(d) Now put the minus 200 back as it was but change the 0.4 in the rule for species A to 0.7. What happens to the populations now? What do you predict for their future?