Final exam

December 15, 1997

Your Name: \_\_\_\_\_

- 1. There are NINE problems. Two of them are on the back of this sheet.
- 2. Write your answers to the nine problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.
- 3. There is scratch paper at the bottom of the last page, but I won't look there for an answer unless you direct me to do so with a note where the answer belongs.
- 4. You may use a calculator.
- 5. You may use notes which you have brought on a sheet of paper, as described in class.

#### Problem 1:

(a) Express each of the numbers 84 and 90 as a product of prime numbers.

- (b) Find the Greatest Common Factor GCF(84, 90).
- (c) Find the Least Common Multiple LCM(84,90).

## Problem 2:

(a) Write  $\frac{5}{7}$  as a repeating decimal. (You can use a calculator to check your work, but you should show the steps necessary to find the answer without a calculator.)

(b) Write  $1.2\overline{35}$  as a rational fraction  $\frac{a}{b}$  for some whole numbers a and b.

## Problem 3:

Two numbers x and y satisfy:

 $\quad \text{and} \quad$ 

x is one more than three times y

two x plus y is thirty.

What are the numbers?

## Problem 4:

For each of the following statements, tell if it is a true statement when applied to the set of all real numbers.

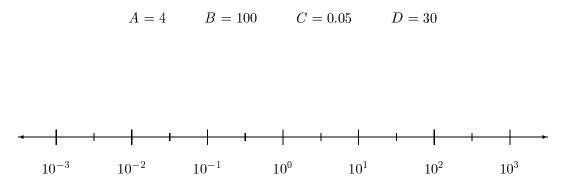
(a)  $\forall x \exists y, x + y = 2.$ 

(b)  $\exists x \ \forall y, \ x \cdot y = 0.$ 

(c)  $\exists x \ \forall y, \ x+y=0.$ 

## Problem 5:

On the logarithmic scale below, mark the approximate location of each number:



## Problem 6:

(a) Convert  $4324_{five}$  to base ten.

(b) Convert  $179_{ten}$  to base three.

(c) Add these two numbers in base seven:

	3	2	4	3
+	<b>2</b>	4	4	2

## Problem 7:

We use the rule  $a \odot b = a+b-5$  to define an operation  $\odot$  on the integers  $\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ . (a) With the operation  $\odot$ , is  $\mathbb{Z}$  closed?

(b) Is the operation  $\odot$  on Z commutative?

(c) Is there an identity element i such that  $a \odot i = i \odot a = a$ ,  $\forall a \in \mathbb{Z}$ ? If so, what is i? If not, why?

## Problem 8:

If  $A = \{1, 2, 3, 5, 6, 7\}$  and  $B = \{1, 3, 7\}$ , what are the following sets?

(a)  $A \cup B$ 

(b)  $A \cap B$ 

(c)  $A \cap \overline{B}$ 

## Problem 9:

If a number is divisible by 3 and is greater than 7 it is a Grub. Classify each of the following statements as True, False, or Can't Tell, and explain.

(a) 12 is a Grub.

(b) 6 is a Grub.

(c) There are more than one hundred Grubs.

# Scratch Paper