Final (takehome) Exam Due at or before 8:00 AM, Sunday, December 14

Do the following six problems as your own work. You may refer to the textbook for assistance. You may ask me for help (which I may or may not give!) but do not work with others.
These problems come from the textbook, but I have reworded some to try to clarify exactly what you have to accomplish. If refer you to the exercises as they appear in the text you may find additional hints for some problems.
Please write your answers legibly, which includes using ink or pencil lead that is dark enough to see easily, and be sure to include your name! Please staple the pages together, do not just fold a corner and please don't use a paper clip! (Paper clips tend to gather up other people's papers...)
Where you are asked to prove something like "Proposition 3.5", or to solve an exercise which is stated in that form even if I have reworded it, you may make use of other propositions and theorems which come before that proposition in the textbook, but not ones that come later.
You may assume the axioms for neutral geometry on all problems: Those are the incidence, betweenness, and congruence axioms from the textbook and a continuity axiom such as the Circle-Circle Continuity Principle or even Archimedes' Axiom together with Dedekind's Axiom, except that you should not use the ability to assign measurements to segments or angles where it is possible to do without measurement. See comments with the problems. If you cannot do without measurement but it is possible to do so, expect to receive reduced credit.

Problem 1: (Exercise 3, page 146)
(There is a typo in the book's statement of part (a), corrected here. Be sure to use the definition of a segment (page 109) rather than an idea derived from an intuitive picture of the plane. Also be careful, a problem on the midterm exam looks similar but it dealt with rays rather than segments. This problem definitely does not require measurement.)
Suppose we have three points A, B, and C satisfying $A * B * C$:

(a) Prove $AB \subseteq AC$.
Prove $CB \subseteq CA$.

(b) Prove $AC \subseteq AB \cup BC$.

(c) Prove $AC = AB \cup BC$.

(d) Prove $B \in AB \cap BC$

(e) Prove B is the only point in $AB \cap BC$, thereby finishing a proof of Proposition 3.5.

Problem 2: (Exercise 9, page 148)
Given a line l and a point A on l and a point B which is not on l: Let P be any point on the ray $\overrightarrow{AB}$ other than the point A.
Show that P must be on the same side of l as B. You may wish to use an argument based on contradiction (RAA). You might wish to consider two cases, based on the definition of a ray on page 109. You will certainly need to use the definition of "on the same side". This problem does not need measurement.

Problem 3: (Exercise 8, page 194)
The goal is to prove Proposition 4.5, page 168, that in a triangle in a neutral geometry the greater side is opposite the greater angle and vice versa. Note that "greater" here does not refer to measurement but to the definitions on pages 124 and 128. You do not need to use measurement in this problem.
The book has an outline of a proof. Many steps need to be filled in. Why do we say "If $AB > BC$"? What does that amount to and why can we say it? What is it we need to show after making that

assumption about the angles? Why must the point D on $\overleftrightarrow{AB}$ exist, and why is it between A and B? Why is triangle ΔCBD isoceles? Which sides are congruent? That says two angles are congruent: Which ones? How do we get $\measuredangle ACB > \measuredangle A$? Why does that finish half the proof?
Going the other way (the "vice versa" part), the book has the cryptic remark "Use this result and trichotomy of ordering...": Explain this and why it finishes the proof, or else construct a separate proof of your own going in the other direction.

Problem 4: (Exercise 10, page 195)
The exercise has two parts: Prove Proposition 4.7, and then deduce another conclusion from it. Be sure you are using the correct version of a parallel postulate, either from the green sheet or from page 138 in the text. In proving Prop. 4.7, be sure to prove both directions of the equivalence: You need to show both that the parallel postulate implies that whenever l and m are some two parallel lines and a line n intersects one of them, say l, then it must also intersect the other one (m), and also the reverse implication. After you have accomplished that, you have to show that either of the two things you have just shown equivalent is also equivalent to (remember that goes two directions!) "transitivity of parallelism": That means if line r is parallel to line s and s is parallel to line t, then either r is parallel to t or $r = t$. (An equivalent definition is given on page 93.)
Does not require measurement.

Problem 5: (Exercise 13, page 195)
The book tersely says "prove Proposition 4.10". Again the problem requires you to prove a property is equivalent to the parallel postulate given on the green sheet: Remember that "equivalent to" requires showing each implies the other, i.e. it goes in two directions. In this case you might want to use one or more of the propositions appearing immediately before 4.10, to avoid reproving one of them.
Does not require measurement.

Problem 6: (Exercise 3, page 228)
Note that this problem definitely assumes we can measure angles. The statement of Wallis' Postulate (page 216) requires "similarity" of triangles, defined on page 215.

(a) The text, on pp. 216-217, proves that Wallis' Postulate implies the Euclidean parallel postulate: Here you are to prove the opposite implication. (But you might find it useful to study the proof the book has, being careful to remember that it does not do the direction you need.) So you have to prove that if we know that through any point outside a line there is one and only one line parallel to the given line, then given any triangle ΔABC and any segment DE, we can construct a triangle ΔDEF which is similar to triangle ΔABC. (I.e. we can make similar triangles of any given size.)

    Proposition 4.11 might be useful here: That and other results in chapter 4 are definitely appropriate to make use of.

(b) You are to show that a modified version of Wallis' Postulate, where we replace similarity by congruence, must be true, not needed as an additional assumption, in any neutral geometry. But be careful, in part (b) you should not assume the Euclidean parallel postulate, since this proof should work in neutral geometry.

    (This may seem strange: Congruence seems to be a stronger property that should somehow be harder to achieve, so how can that be the one that works without assuming a parallel postulate? But the actual statement of Wallis Postulate includes the implication "of any given size" which is why it requires more.)