Math 340 Lectures 1 and 2, Spring 2011 — Wilson

Topics for first midterm exam
(Exam: 5:30–7:00 PM, Thursday, February 24, in B130 Van Vleck)

In general problems will be similar to suggested problems listed on the class schedule. You definitely will be expected to be answer some problems requesting proofs. I will not ask you to recite complicated proofs from lecture or from the textbook, but some of the shorter proofs that we have seen are possibilities. I'm not sure how to quantify that, but as examples I think the proof in the book (or from class) of Theorem 1.6 (page 48) would be perfectly reasonable, while the proof of Theorem 2.2 (page 92) is too complicated to expect you to explain as an exam problem. I could also ask you to prove something that has not been stated as a theorem, if it were very simple and I either gave you a hint or there was a clear reason why it should be true. There won't be any problems on the specific applications described in the text, such as "Quadratic Interpolation" starting on page 103.

You are allowed to use a calculator, including graphing calculators, but not a portable computer or any other device that can browse the web. But there are several things you can infer from access to a calculator: (a) I will try to make the exam pretty "calculator neutral", so that someone with an expensive calculator has not bought much of an advantage over someone who does not have that. (b) In grading we will be concentrating on your showing how you get your answers rather than the numbers making up those answers. So be sure to show your work and to explain why you are doing things. Don't put in lots of extra words just to make the answer longer, but do make clear what you are doing.

You are allowed to bring in one "note card": It should not be bigger in area than one half of an ordinary sheet of typing paper, $8\frac{1}{2}$ inches by $5\frac{1}{2}$ inches. (A large index card, 6×8 inches, would be OK.) On that card you can write anything you want, you can computer print it in tiny letters, use both sides, what ever.... If I were doing it, I would just put those things I was most worried about either forgetting or getting mixed up: If you were not sure which permutations got + signs, that might be something to put on the card. If you try to reconstruct the whole course on there you will spend the entire exam time searching for things.

The exam will have N problems, for some $7 \leq N \leq 10$, and there will be space on the exam both for you to write your answers and for some scratch work. (We will have some extra paper in case you need more space for either purpose.) So all you need to bring is something to write with and your knowledge, and a calculator if you choose.

§1.1 Systems of Linear Equations

Any system for which we ask you to find the answers will be given in matrix form, so you don't have to translate between a set written out as equations and their matrix form. But you do need to know what the equations mean, so that for example you could tell whether a given sequence of numbers was a solution. You should be familiar with the terminology, e.g. consistent vs. inconsistent, trivial vs. nontrivial solutions, and equivalence of systems of equations.

§1.2 Matrices

I suppose it is no surprise that you should know what a matrix is! You should know the terminology and notation: Which subscripts correspond to the i^{th} row or column? Where in a matrix A is the element a_{32} found? Although we have sometimes referred to the $m \times 1$ matrix $\vec{b}$ consisting of the "right hand sides" of a system of equations as a vector, I have put off using the word vector much until we get to talking about vector spaces, so that particular term is not yet important for the exam.

Be able to define when two matrices are equal. This section defines the basic operations with matrices, and of course you need to know how to perform those, but it may make more sense to merge this with the section §1.4 which talks about many properties satisfied by those operations.

This section introduces the term "linear combination". Just as with the word vector I have been putting that off until we encounter vector spaces.

§1.3 Matrix Multiplication

Just as with §1.2, the book defines the operation and puts off properties until §1.4. But the matrix product is sufficiently complicated that it is worth spending time on just how it is computed. Be sure you can multiply matrices! This section also introduces the terminology “coefficient matrix” and “augmented matrix” we use when dealing with a set of equations in matrix form, and defines a homogeneous system. Be sure you are familiar with these terms.

§1.4 Algebraic Properties of Matrix Operations:

You must be familar with these! Remember what properties the operations don’t have, also, e.g. the facts that matrix multiplication is not commutative, can give a product that is zero (the zero matrix) even when neither factor is zero, and does not generally allow for cancellation. (These particular properties are highlighted in blue on page 39.)

§1.5 Special Types of Matrices and Partitioned Matrices

You certainly need to know the terms “symmetric” and “skew symmetric”. The exam will not require you to work with partitioned matrices or block multiplication. But don’t lose sight of the fact that singular, nonsingular, and inverse of a matrix are all defined in this section, and several related properties appear as theorems in this section.

§1.6 Matrix Transformations

This section, as well as sections §1.7 and §1.8 on particular applications, will not be on the exam!

§2.1 Echelon Form of a Matrix

I will not ask you to recite the definition of Row Echelon Form or Reduced Row Echelon Form (which may appear on the exam abbreviated as REF or RREF), but you definitely need to be able to tell if a matrix is in REF or RREF. You need to know what are the legal elementary row operations: I will not assume you have labelled them as Type I, etc. You should know when two matrices are row equivalent. The proofs of Theorems 2.1 and 2.2 are really descriptions of how to find elementary row operations that could be applied sequentially to a matrix A to transform it to a matrix in REF or RREF, but (a) there would be many such sequences, not just the ones outlined in the proofs, and (b) while the matrix you would get in RREF is unique in the sense that only one such matrix exists for one starting matrix, that is not true for REF. So starting from an $m \times n$ matrix A you should be able to apply elementary row operations resulting in REF or RREF as requested, but someone else might have a different set and for REF someone else might get to a different matrix at the end.

§2.2 Solving Linear Systems

If you have the augmented matrix for a system of equations in REF or RREF, you should be able either to write a description of all solutions (be they one or many) or tell that there are no solutions of the corresponding equations. Exam problems are not really like the real world (in case you had not noticed!), so there might be separation between problems related to getting to REF or RREF and problems where the matrix was given to you already in a nice form. While you should be able to carry out both Gaussian elimination and Gauss–Jordan reduction, including back substitution, you do not need to worry about those names. But do note the general discussion of Homogeneous Systems, including Theorem 2.4, and the material on the relation between Homogeneous and Nonhomogeneous systems, that sneaks in on pages 108-111.

§2.3 Elementary Matrices; Finding A^{-1}

By now we have so thoroughly integrated elementary row operations and elementary matrices that you might have forgotten that there was actually a lot of the textbook in between them. That’s fine. I think it makes sense to keep them together now that we have them. But be sure you do know the material relating them in Theorems 2.5 and 2.6, as well as the properties of elementary matrices in the theorems, lemma, and corollary in this section. After those properties there are more results (This is a really packed section!): The blue shaded box on page 120 nicely pulls some things together, Theorems 2.10 and 2.11 are very important, and the blue boxes on pages 123 and 124 simplify some procedures.

§2.4 Equivalent Matrices

Notice this means a more general equivalence, allowing column as well as row operations: This section will not be on the exam, and §2.5 is also skipped.

§3.1 Definition (of the Determinant)

You <u>should</u> be able to:

- Tell whether a permution is odd or even
- Write out as the defining sum the determinant of a 2×2 or 3×3 matrix
- Attach the correct sign ($+$ or $-$) to a term from the determinant sum for a somewhat larger matrix: E.g., which sign goes with $a_{15}a_{23}a_{36}a_{41}a_{57}a_{64}a_{72}$ in the determinant of a 7×7 matrix?

You won't need to write out all $n!$ terms and compute a determinant from the definition, for $n \geq 4$.

§3.2 Properties of Determinants

You need to know and be able to use the properties in Theorems 3.1 through 3.9 and Corollaries 3.1 through 3.3. You should be able to find the determinant of a matrix using "reduction to triangular form" or to any other form that is convenient for you, a form where the determinant is easily computed. (E.g. if you are energetic and want to go beyond triangular to REF, that would be OK, just likely to be extra work.) But be sure that whatever form you aim for, you (a) only apply legal operations and (b) you properly "correct" for the effect of those operations.

Be sure you understand how elementary row operations affect the determinant well enough to use these facts on the exam: Over the years I have seen students get this wrong, when they try to work back from something (e.g. I_n or a triangular matrix) where the determinant is easily computed, to get to the determinant of the matrix they started with. If, e.g., you multiply a row by 3, that makes the determinant of the <u>new</u> matrix 3 times the determinant of what you had before, so the determinant you were trying to find is $\frac{1}{3}$ of the determinant of the new matrix. That is "obvious" if you think about it, but easy to get backwards working under the pressure of an exam! (I.e., to mix up whether you should multiply by 3 or by $\frac{1}{3}$ in your final calculations.)

Notice that Corollary 3.3 uses the property of "similarity" of matrices, although that is not officially defined for almost 300 pages! So you need to know what it means for A and B to be similar (see page 410) in order to understand that corollary, but once you know what it means the corollary is easy to prove.

§3.3 Cofactor Expansion

You should be able to evaluate minors and cofactors, and to use cofactor expansion along any convenient row or column to evaluate a determinant (the content of Theorem 3.10). (The very first fact in §3.4, not on the exam, will (a) be covered in lecture before the exam and (b) looks much like Theorem 3.10 with an important difference. Be sure you know what Theorem 3.10 <u>does</u> say, and how to use it, so you don't get confused.)