Your Name:			
Please circle your TA's name:	Jaeho Lee	Andrew Bridy	
Mathematics 340, Spring 2011	Lectures 1 & 2 (Wilson)		
Second Midtorm Exam	April 14 2011		

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There is a problem on the back of this sheet: Be sure not to skip over it by accident!

There is a sheet of scratch paper at the end of this exam. If you need more scratch paper, please ask for it.

You may refer to notes you have brought on part of a sheet of paper, as announced previously.

In proving something you may cite any theorems, lemmas, or corollaries from the text, so long as they came <u>before</u> what you are proving. I.e., don't indirectly use something to prove itself!

The symbol \vec{x} means	$\begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}$	$\left],ec{b} ext{means} ight.$	$\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$, and $ec{0}$ means	$\begin{bmatrix} 0\\0\\\dots\\0\end{bmatrix}$, for whatever size is
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appropriate.

You may assume that vector spaces \mathbb{R}^n , \mathbb{R}_n , M_{mn} , and P_n (with their usual operations), that we have frequently used, are vector spaces, without further proof. Do not assume other spaces are vector spaces unless specifically told to do so.

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS.

Problem	Points	Score
1	10	
2	12	
3	10	
4	12	
5	10	
6	10	
7	12	
8	12	
9	12	
TOTAL	100	

<u>Problem 1</u> (10 points)

(a) Consider the matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, as vectors in the vector space M_{22} of all 2×2 matrices.

Do these matrices span M_{22} ?

Be sure to give reasons for your answer!

(b) Now use the matrices
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
, $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. Are these four vectors in M_{22} linearly independent?

Again be sure to give reasons!

(Note that some of these matrices are the same as in part (a), if that is any help.)

<u>Problem 2</u> (12 points)

Let $S = {\vec{v_1}, \vec{v_2}, \dots, \vec{v_k}}$ be a set of k vectors in some vector space V. Prove that S is linearly dependent if and only if (at least) one of the vectors $\vec{v_j}$ can be written as a linear combination of all the other vectors in S.

(Don't overlook the word "all"! Also, don't use Theorem 4.7, which stated that a set was linearly dependent if and only if some vector was a linear combination of preceding vectors, in your proof. But the ideas that were in the proof of Theorem 4.7 would certainly be OK to use. This was one of the homework problems, I have slightly revised the wording to try to make clearer exactly what is going on.)

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Problem 3 (10 points) Let $v_1 = \begin{bmatrix} 1\\1\\0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2\\2\\0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1\\-1\\1 \end{bmatrix}$, $v_4 = \begin{bmatrix} 1\\0\\1 \end{bmatrix}$, and $v_5 = \begin{bmatrix} 5\\5\\2 \end{bmatrix}$. Find a basis for \mathbb{R}^3 consisting of some of the vectors $\vec{v}_1, \dots, \vec{v}_5$.

(If you happen to do problem 6 before this one, you might find useful some calculations done for that problem.)

Let V be any vector space. Suppose for some vector \vec{u} in V and some number $c, c \odot \vec{u} = \vec{0}$, where \odot is whatever the scalar multiplication is on V and $\vec{0}$ is whatever is the zero vector for the space V.

Prove that either c = 0 (the real number 0) or $\vec{u} = \vec{0}$.

(This is part (c) of Theorem 4.2: Parts (a) and (b) were that $0 \odot \vec{u} = \vec{0}$ and $c \odot \vec{0} = \vec{0}$ for any vector \vec{u} and for any number c. You may use those facts in your proof for this problem if you find them helpful.)

<u>Problem 5</u> (10 points)

(a) Show that the set of vectors $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in M_{22} which satisfy a + b + c + d = 0 is a subspace of M_{22} .

(b) Show that the set of vectors $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in M_{22} which satisfy a + b + c + d = 1 is <u>not</u> a subspace of M_{22} .

<u>Problem 6</u> (10 points)

Let
$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 5 \\ 1 & 2 & -1 & 0 & 5 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$
.

(You may find useful some work done in connection with problem 3.)

(a) Find a basis for the null space of A, the space of solutions of $A\vec{x} = \vec{0}$.

(b) What is the rank of A?

Problem 7 (12 points)

For the vector space $V = P_2$ of polynomials with degree at most 2:

- (1) The set $B_1 = \{1 + t, 1 + t^2, t^2\}$ is a basis for V. (2) The set $B_2 = \{t, 1 + t, 1 + t^2\}$ is also a basis for V.
 - (a) Find the "change of basis" matrix (the matrix that the book calls $P_{B_1 \leftarrow B_2}$) that tells how coordinates with respect to B_2 change to become coordinates with respect to B_1 .

(b) Find the coordinates of $1 + t - t^2$ with respect to B_2 .

(c) Find the coordinates of $1 + t - t^2$ with respect to B_1 .

(d) As a check on your work, multiply the matrix you got in (a) on the left of the coordinate vector resulting from (b) and show that you get the vector corresponding to (c).

<u>Problem 8</u> (12 points)

Let
$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$
.

Hint: You might find (b) and (c) easier if you use the result from (a).

(a) Find the adjoint adj(A).

(b) Find the determinant det(A)

(c) Find the inverse A^{-1} .

 $\frac{\text{Problem 9}}{\text{Prove that the only subspaces of } \mathbb{R}^2 \text{ are}$

- 1. The subspace consisting of just $\begin{bmatrix} 0\\0 \end{bmatrix}$
- 2. The subspace consisting of all multiples of some non-zero vector \vec{v} , and
- 3. The whole space \mathbb{R}^2 .

Hint: Consider cases. If V is a subspace of \mathbb{R}^2 , is there anything non-zero in it? If there is, are there any two vectors in V that are linearly independent? Could there be more than two vectors in V that were linearly independent?



Scratch Paper