Math 222 Lecture 1, Spring 2008

Final Exam notes and topics

The final exam will be cumulative, covering topics from the entire course. Since there is little that has been covered since the second midterm and thus has to get special emphasis, about 80% of the exam will be on earlier material: This gives you the opportunity to show us you know how to do something even if you messed it up on an earlier exam. You will have two hours: I will try to make it not much longer than the midterm exams, for which you had 90 minutes.

Here is a list of topics, with comments. Note that there certainly cannot be a question on every topic, but also that some questions may cover more than one item. For example, computing area in polar coordinates also involves evaluating an integral, which might invoke material from the sections on techniques of integration.

You may use a calculator, and you may bring notes on up to three notebook-paper-sized sheets. The formulas for integrals that were included on the first exam will <u>not</u> be included on this exam (in order to save paper) so if you feel you need some of those you should include them in your own notes. You may bring the yellow sheet on undetermined coefficients (it does not count as part of your notes) if it does not have additional material written on it.

- Techniques of integration
  - You certainly can't get far without being able to use substitution, but there will be no question centered on just that.
  - Integration by parts: An essential tool. Be sure you know how to use it in cases where it gets used more than once, or perhaps used a couple of times and then the answer is solved for algebraically.
  - "Rational Functions", aka Partial Fractions: There will not be a problem <u>requiring</u> this technique, but you are welcome to use it if that is how you want to do some problem.
  - Trignometric Integrals: Integrals involving products or quotients of powers of $\sin\theta$ and $\cos\theta$, or of powers of $\tan\theta$ and $\sec\theta$, and other integrals evaluated using trig identities. You certainly need to be able to use $\sin^2\theta + \cos^2\theta = 1$ and $\tan^2\theta + 1 = \sec^2\theta$ to carry these out. But you can expect all the trig functions in any one integral to have the same argument, i.e. not like $\sin 2\theta \cos 3\theta$.
  - "Trigonometric substitutions": The text consistently does this using formulas, I consistently draw a triangle to establish the relationships. You are welcome to go either way. But be sure you can handle definite integrals correctly.
  - Numerical Integration: There will not be a problem requiring this.
  - Improper Integrals: Be sure to notice if the integrand "blows up" and use limits both for that situation and for the case where an endpoint is infinite.
    <u>Improper integrals evaluated without explicit attention to limits will receive no partial credit</u>.
- Conic Sections and Polar Coordinates
  - You should be able to find equations for conic sections given a geometric description, or to establish facts (e.g. foci, where it crosses the axes, asymptotes, directrix for a parabola) about a conic section from an equation. You will not be asked to draw the graph for a conic section, but you might be asked questions about a graph.
  - There will not be a problem on translation or rotation of axes.
  - You should be able to work from a parametrization to what path it follows and, in reverse, to parametrize motion along a curve.
  - You should be able to convert between rectangular and polar coordinates.
  - You should be able to recognize and to sketch the more common polar curves such as circles and cardioids. You will not be asked to plot a polar curve and graded on your drawing, but you may

find a sketch useful in order to find tangent lines or area. But if you need to find a tangent line or area for a very complex polar curve, I would probably give you at least the shape of the graph.

  - Polar equations for conic sections will require at most the standard forms for lines (page 732) and for a conic with eccentricity e (page 734), together with interpretation of eccentricity (§10.2).

- Sequences and Series
  - Infinite sequences, and series of constants:
    * Know what it means for a sequence or series to converge, and be able to apply that definition. Be sure you know the difference between a sequence and a series!
    * Be familiar with some special classes of series: Geometric series, harmonic and alternating harmonic series, and p-series. Know when they converge or diverge. For geometric series you should know how to calculate both the sum of the first n terms and the sum of the series.
    * Be able to test series for convergence/divergence. You should at least be able to use the n^{th}-term test, the ratio test, the comparison test, the integral test, and the alternating series test. If you use other tests correctly that will be OK, but for any test be sure to make clear how you know its requirements are met.
    * Be able to distinguish between absolute and conditional convergence as well as divergence. This at first applied to a series of numbers, but as a consequence it applies to a power series if values of the variable are given.
  - Power Series:
    * Be able to find the interval of convergence for a power series, including testing at end points.
    * You should be able to use sums of series, geometric series, integration, and differentiation as tools to find power series for given functions or to go from a series to a function it represents. I.e., don't have Maclaurin and Taylor series as your only ways for finding power series!
    * You should know the Maclaurin series for $\cos(x)$, $\sin(x)$, and e^x, at least.
    * You should be able to find the first few terms of the Maclaurin or Taylor series for a function, and to give a formula for the n^{th} term if there is a clear repetitive pattern among the derivatives. In general you will have to show how you got the terms of the series, not just read them from a fancy calculator, but you do not have to re-derive the series such as e^x referred to in the previous item.
    * You should be able to work with the "error" resulting from using only initial terms of a series. This implies the ability to use the remainder term $R_n(x)$ from Taylor's theorem. In some but by no means all cases you can save some work by using the remainder term from the alternating series test. You should be able to bound the error for a fixed number of terms as well as to find the number of terms needed to achieve prescribed accuracy.

- Differential Equations
  - There will not be any "story problems" on differential equations like mixture problems or spring/mass or electric circuit problems.
  - You should be able to plot a solution on a slope field.
  - You should be able to solve separable first order differential equations and initial value problems.
  - You should be able to solve first order linear differential equations and initial value problems.
  - There will not be a problem requiring use of Euler's method for numerically solving a differential equation.

  - You should be able to solve second-order, linear, constant-coefficients differential equations and initial value problems.
    This includes both homogeneous and nonhomogeneous cases: You may assume any nonhomogeneous case that you have to deal with can be handled using Undetermined Coefficients, but if you wish to use Variation of Parameters that is certainly OK. You may bring with you the class handout on use of Undetermined Coefficients, with its table of suggested trial functions, so long as you have not written additional notes on it.
  - There will be no problems requiring power series solutions to differential equations, or Euler equations, such are found in §17.3-17.5.

- 3-D Coordinates and Vectors
  - You should be able to do algebra with vectors, including finding the vector from one point to another, adding vectors, representing them in different ways, calculating length, multiplying by constants, finding a unit vector, and calculating dot and cross products. You should be able to express a given vector in terms of its magnitude and a direction vector. You should be able to find the angle between two vectors or between two planes or between two lines, and in particular to tell whether they are orthogonal. You should be able to express a given vector as a sum of two components, one parallel to and the other orthogonal to, another given vector. You should be able to calculate the area of a triangle or parallelogram in space, given vectors along two non-parallel sides or given the vertices. (There are several ways to write out the cross product calculations: You should be able to carry out some such calculation by hand. I.e., you must be able to show how the cross product is calculated and "I got the answer from my calculator" is not good enough.)
  - You should be able to use vectors in geometry, finding scalar and vector projections, distance from a point to a line or plane, equations for planes described geometrically, parametric forms of equations for lines or line segments, the line in which two planes intersect, and similar applications of vector algebra.
  - You should be able to match an equation with a cylinder or quadric surface, and to identify a surface (cylinder, paraboloid, ellipsoid, etc.) from the equation or from a picture.