Topics For Second Exam
Math 222, Lecture 1, Spring 2008

Earlier material of course is still relevant, e.g. one could not perform an integral test on a series without being able to evaluate an improper integral properly. But here is a list of the topics that will be specifically targeted, with some comments also on topics that will not appear on the exam.

- §10.7: You should be able to find the area of a plane region described using polar coordinates. This can include finding where polar curves intersect and figuring out what a region looks like, with enough detail to set up correctly one or more integrals. You will not be asked to compute arc length, or surface area for surfaces of revolution.

- §10.8: You should be able to use the polar equation for a conic to determine whether a given equation corresponds to ellipse, parabola, or hyperbola, but there will be nothing else from this section.

- §11.1 and 11.2: First and foremost, be careful about the distinction between a sequence and a series. You should be able to use the definition for convergence of a sequence and also the definition for convergence of a series. You should be able to take the first few terms of a sequence or series with an evident pattern and either find some more terms or write a rule for the n^{th} term, and to deal with a sequence or series where the terms are described recursively. You should be able to use the Nondecreasing Sequence Theorem.
  Geometric series are introduced in §11.2: You should be able to work fluently with them. A telescoping series problem is possible but it would have to use very simple algebra. Be sure to know what the n^{th}-term test does say and what it does not say!

- §11.3-11.5, tests for convergence of positive series: Any exam problem will be doable using the integral test and/or the ratio test and/or the comparison test. (I.e. some tests such as limit comparison and root won't be needed.) But be sure you know what those tests require, and if you apply one be careful to point out that it really does apply by checking that the conditions are met. (E.g., for the integral test, that the function you fit to the terms really is positive and continuous and decreasing beyond some clearly stated point.) If you can use more tests then you have a better chance of finding an easy way to do a convergence problem. You definitely should know some series that converge and some that diverge to use for comparisons.

- §11.6: Be sure you know the conditions under which Leibniz' Theorem applies, and if you use it that you check that the conditions are satisfied. Be able to use the "Alternating Series Estimation Theorem" to bound the error in using just part of a series and also to tell whether that part of the series gives too big or too small an estimate of the actual series sum.
  You should know how to tell if a series is absolutely convergent, conditionally convergent, or divergent. You should be able to use absolute convergence to show that a series with some negative and some positive terms (not necessarily alternating) converges, referring to the tests for positive series.

- §11.7: You need to know what a power series is and how it must converge (corollary on page 798) and how to find the radius and the interval of convergence. You should be able to use term-by-term differentiation and integration to convert one power series to another, knowing where the result will converge, or to recognize what function some given series would converge to. You will not be asked to multiply series by series, but you should be able to deal with multiplying a series by a power of x or by a simple polynomial. Don't assume that all power series are found using Maclaurin or Taylor series!

- §11.8: You definitely do need to be able to find a Maclaurin or Taylor series for a given function. This includes finding a pattern that lets you tell what the n^{th} term would be. Taylor polynomials are introduced here but are really not useful until the next section.

- §11.9: This is the most important section in the chapter, for almost any application most of you might someday make of this material. If you don't know how accurate something is, you have no business using it. You should be able to do three different kinds of problems where you estimate the error that results from substituting the n^{th} degree polynomial $P_n(x)$ (the first $n+1$ terms of a Taylor or Maclaurin series) for the entire series that would give $f(x)$ if we could sum the entire series: (a) If we are told to use the polynomial of a given degree at a prescribed point, how bad could the error be? (b) If we use a given degree but the variable x can vary over some interval, what is the worst the error could be? (c) If we need a certain accuracy and the variable could be anywhere in some given interval, what n should we use to get the required accuracy?
  While Euler's Identity appears in this section, there will be no direct question on it: We will use it in §17-1 so you will need to know it for that.

- §11.10-11.11: There will be no questions directly on this material. But the table on page 831 does have a list of series it would be good to know in order to save time.

- §4.8, §9.1: Of course you need to know terminology (What is a differential equation? An initial value problem?) in order to read the exam problems. While being able to infer a solution from a slope field is very useful, problems like that lead to arguments about grading so I will not ask you that. You definitely need to be able to solve separable differential equations and corresponding initial value problems.

- §9.2: You should be able to solve first-order linear differential equations and initial value problems. Note that you might have to rearrange an equation before it was in the "standard form" assumed by the machinery developed in this section. If there is a "story problem" on this (or for that matter on any other topic) (a) it must be one that involves fairly common concepts like motion or mixing solutions and (b) it must be fairly short and simple.

- §9.3: If we ask you an Euler's Method problem it will (a) involve few sub-intervals and (b) use functions such that the arithmetic works out easily.

- §9.4, §9.5: No exam problems

- §17.1: You have to know the terminology (e.g. "homogeneous"). You should know how to combine solutions of an homogeneous equation to get others (the superposition principle). You need to know how to find two linearly independent solutions to a second-order homogeneous linear equation with constant real coefficients. That comes down to three different situations according to whether the characteristic equation has two distinct real roots, a double real root, or a pair of complex conjugate roots: In the latter case you should be able to write the solutions in terms of trig functions. As always with differential equations you should be able to solve related initial value problems: In this case there will be two constants to solve for. (The book makes a distinction between initial value and boundar value problems, at the end of this section. That distinction won't matter for us, you should treat them alike.)

- §17.2: You need to be able to find one solution of a non-homogeneous, second-order linear, differential equation with constant coefficients, and to combine that with all the solutions to the associated homogeneous equation to find all solutions to the non-homogeneous equation. You are welcome to use either Undetermined Coefficients or Variation of Parameters to find the particular solution, and any exam problem will be doable with either method.

- §17.3: This section is mostly just examples using material from §17.2 in different story problems: I will not ask you to set up and solve any of these on the exam. (It might help you to understand what is going on in chapter 17 if some of those story problems can be related to application subjects you are interested in, though.) But you should understand the difference between different degrees of damping, i.e. what the solutions are like when $\sqrt{b^2 - \omega^2}$ is positive, negative, or zero.

- §17.4: Skipped

- §17.5: Use of power series in solving differential equations. This will not be on the exam.