

## Math 222 Lecture 4, Spring 2005

### Final Exam topics

The final exam will be cumulative, covering topics from the entire course, with somewhat extra weight being given to the material since the second midterm exam. Here is a list of topics, with comments. Note that there certainly cannot be a question on every topic, but that on the other hand some questions may cover more than one item. For example, computing area in polar coordinates also involves evaluating an integral, which might invoke material from the sections on techniques of integration.

- Techniques of integration
  - You certainly can't get far without being able to use substitution, but there will be no question centered on just that.
  - Integrals involving products of powers of sin and cos, or products of powers of tan and sec, and other integrals evaluated using trig identities.
  - "Rationalizing substitutions": The text consistently does this using formulas, I consistently draw a triangle to establish the relationships. You are welcome to go either way.
  - Integration by parts: An essential tool. Be sure you know how to use it in cases where it gets used more than once, or perhaps used a couple of times and then the answer is solved for algebraically.
  - "Rational Functions", aka Partial Fractions: There will not be a problem requiring this.
- Chapter Nine
  - L'Hôpital's Rule: There will be no problem specifically on this, which many of you had in 221, but you might well need to use it. For example, it can help find the limit used in a ratio test for series.
  - Improper Integrals: You should expect to see this on the exam. Remember how to deal with an integral improper within the interval of integration, where no tell-tale  $\infty$  appears on the integral! Just evaluating an indefinite integral where the function blows up inside the interval, by antidifferentiating and plugging in the end points, without explicitly using limits, will get zero credit even if the answer happens to be the right number.
- Sequences and Series
  - There will be no problem explicitly on sequences, but you need to be able to work with them to understand and in some cases to evaluate the sum of a series.
  - Infinite series of constants:
    - \* Know what it means for a series to converge, and be able to apply that definition.
    - \* Be familiar with some special classes of series: Geometric series, harmonic and alternating harmonic series, and  $p$ -series. Know when they converge or diverge. For geometric series you should know how to calculate both the sum of the first  $n$  terms and the sum of the series.
    - \* Be able to test series for convergence/divergence. You should at least be able to use the  $n^{\text{th}}$ -term test, the ratio test, the comparison test, and the alternating series test. Some problems may go more quickly if you can also use the integral test and the limit comparison test.
    - \* Be able to distinguish between absolute and conditional convergence as well as divergence.
  - Power Series:
    - \* Be able to find the interval of convergence (convergence set) for a power series, including testing at end points.
    - \* You should be able to use geometric series, integration, and differentiation, as tools to find power series for given functions or to go from a series to a function it represents.
    - \* You should know the Maclaurin series for  $\cos(x)$ ,  $\sin(x)$ , and  $e^x$ .

- \* You should be able to find the first few terms of the Maclaurin or Taylor series for a function, and to give a formula for the  $n^{th}$  term if there is a clear repetitive pattern among the derivatives. In general you will have to show how you got the terms of the series, not just read them from a fancy calculator, but you do not have to re-derive the series such as  $e^x$  referred to in the previous item.
- \* You should be able to work with the “error” resulting from using only initial terms of a series. This implies the ability to use the remainder term  $R_n(x)$  from Taylor’s theorem. In some but by no means all cases you can save some work by using the remainder term from the alternating series test. You should be able to bound the error for a fixed number of terms as well as to find the number of terms needed to achieve prescribed accuracy.
- Numerical methods
  - You should be able to use the Trapezoidal Rule and Simpson’s (parabolic) Rule to approximate an integral.
  - For reasons of practicality there will be no problems on sections 11.3-11.5.
- Analytic Geometry
  - You should be able to find equations for conic sections given a geometric description, or to establish facts (e.g. foci) about and sketch a conic section from an equation.
  - There will not be a problem on translation or rotation of axes.
  - You should be able to convert between rectangular and polar coordinates.
  - You should be able to recognize and to sketch the more common polar curves. You will not be asked to plot a polar curve and graded on your drawing, but you are likely to need a sketch in order to find tangent lines or area.
- Differential Equations
  - You should be able to solve second-order, linear, constant-coefficients differential equations.
  - This includes both homogeneous and nonhomogeneous cases: You may assume any nonhomogeneous case that you have to deal with can be handled using Undetermined Coefficients, but if you wish to use Variation of Parameters that is certainly OK. The table of suggested trial solutions for Undetermined Coefficients that I gave out in class will be supplied with the exam.
  - You should be able to find the general solution to a differential equation, i.e. a representation of all functions that satisfy the equation. You should also be able to solve an initial value problem, i.e. find the solution that additionally satisfies some initial conditions.
  - There will be no “story problems” as found in section 18.3.
- Parametric Equations and Vectors
  - You should be able to sketch a curve given parametrically and to find the tangent line to it.
  - You should be able to do algebra with vectors, including finding the vector from one point to another, adding vectors, representing them in different ways, calculating length, multiplying by constants, finding a unit vector, and calculating dot and cross products. (There are several ways to write out the cross product calculations: You should be able to carry out some such calculation by hand. I.e., you must be able to show how the cross product is calculated and “I got the answer from my calculator” is not good enough.)
  - You should be able to use vectors in geometry, finding scalar and vector projections, equations for planes described geometrically, equations for lines, and similar applications of the vector algebra.
  - You should be able to find the velocity and acceleration vectors for a given position vector, and the tangent line to a curve traced by a position vector.