Series worksheet **Math 222, Section 1** **Wilson** **February, 2004**

The following extra problems on sequences and series are for you to practice on, in preparation for our first exam. The exam won't consist of just these problems, but facility with these problems should help you do well on the exam. I don't have written-up answers to these, but I will be glad to answer questions about them.

1. If $a_1 = 1$ and $a_2 = 3$ and $a_n = a_{n-1} - a_{n-2}$, what is a_5?

2. Which of the following sequences $\{a_n\}$ converge? If a sequence does converge, what is its limit?

   (a) $a_n = \frac{n^2+3n-1}{4n^2-2}$.

   (b) $a_n = 1 + \sin\left(\frac{n\pi}{2}\right)$.

   (c) $a_n = 1 + \sin n\pi$.

   (d) $a_n = 3 \times \left(\frac{1-(.7)^n}{1-.7}\right)$.

3. Find a formula for the n^{th} partial sum of the series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \ldots$. Then take the limit of that sum as $n \to \infty$ to evaluate the sum of the series.

4. Which of the following series converge? For each series give a reason for your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} 2^{1/n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\cos(n)}{n^2}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{3^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{n\ln(n)}{2^n}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

5. For each of the following series tell whether it converges absolutely, conditionally, or not at all. Justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+3}{1000n^2 - 500n + 5}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+3}{1000n^3 - 500n + 5}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2+3}{1000n^4 - 500n + 5}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(-5)^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$, for $p > 1$.

   (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^p}$, for $0 < p \leq 1$.

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

   (h) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n!}$

6. The series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$ (where the denominators run through the odd numbers) converges to $\frac{\pi}{4}$.

   (a) Although I already said it converges, prove that using some test we have had.

   (b) If we wanted to approximate π we could use the first n terms of that series, for some n, and multiply the sum by 4. What terms should we use if we need to get our value for π correct to within ± 0.01?