Approximation worksheet **Math 222, Section 1** **Wilson** **March, 2004**

I do not think the book has enough problems on approximating a function, or a specific value of a function, using Taylor or Maclaurin polynomials. Other departments who have required many of you to take Math 222 place this as their highest priority in connection with series, for some the only reason they would want you to study series at all. Here are some additional problems to practice on.
This is material many students find difficult. The difficulty probably comes in large part from two things: (a) There is frequently more than one way to do the problem, and in almost all cases there are many (frequently infinitely many) correct answers. This is not the picture people get from K-12 mathematics of how math problems should work, but in fact it is very much the case with most real-world math problems. (b) In estimating the remainder term for Taylor's theorem, you have to think of how big a function could get on an interval rather than how big it actually is at any specified point. That requires stepping back and getting your mind around the function as a function rather than just how to compute it at a point. This ability is actually useful in many places, and many problems students have in material even before this might disappear if this viewpoint were more common, but it is essential here.

1. Calculate e with an error of at most 10^{-7}.
2. If we use $x - \frac{x^3}{3!}$ to approximate $\sin(x)$, for what values of x is the approximation correct to within ± 0.0003?
3. If $\cos(x)$ is replaced by $1 - \frac{x^2}{2}$, for $|x| < 0.5$, estimate the error resulting. Does this approximation tend to be larger or smaller than the actual value of $\cos(x)$ for these x values?
4. The approximation $\sin(x) \approx x$ is sometimes used for small values of x. How good is it, if we only use it for $|x| < 0.001$? For which of those values of x will x be less than $\sin(x)$? greater?
5. (a) Use the remainder term in Taylor's theorem to estimate the error that results if we replace e^x by $1 + x + \frac{x^2}{2!}$ for $|x| < 0.1$.

   (b) For $x < 0$, the series $1 + x + \frac{x^2}{2!} + \ldots$ is an alternating series. Use the Alternating Series estimation process to estimate the error that results if we replace e^x by $1 + x + \frac{x^2}{2!}$ for $-0.1 < x < 0$.

   Compare your answers to (a) and (b): How can they be different and both be correct?
6. (Continuing from our quiz of 3/5/04...) The first two terms of the Taylor series for $\sqrt{x}$ at $a = 1$ are $1 + \frac{1}{2}(x-1) = \frac{1}{2} + \frac{x}{2}$. If we approximate $\sqrt{0.95}$ using that polynomial, how accurate will the results be? Use one of the theorems we have had, and then compare the answers you get from the polynomial and $\sqrt{0.95}$ using a calculator. Are these consistent with the theoretical results?
7. How many terms of the Maclaurin series for $\ln(x+1)$ should you include to be sure of calculating $\ln(1.1)$ with an error of magnitude less than 10^{-8}?
8. Use the Alternating Series test to decide how many terms of the Maclaurin series for $\arctan(x)$ ($\tan^{-1}(x)$) should be included to get $\frac{\pi}{4}$ within 10^{-3}.