Solution in detail to problem 2 on page 492 in Varberg text:

The problem asks us to (a) calculate the Riemann sum corresponding to 8 equal-width subintervals and using the left endpoint of each subinterval to evaluate the function, (b) use the trapezoidal rule with 8 subintervals and (c) use Simpson's (parabolic) rule with 8 subintervals to approximate the integral, and finally (d) use the fundamental theorem of calculus to evaluate the integral, for

$$\int_1^3 \frac{1}{x}\,dx.$$

Since (a), (b), and (c) all use division of the interval $[1, 3]$ into 8 equal parts, let's first work out the details of that partition and then have notation set up for the rest of the problem. The subintervals will each have width $\Delta x = h = \frac{3-1}{8} = \frac{1}{4}$. The first subinterval will start at $x_0 = 1 = \frac{4}{4}$ and extend to $x_1 = 1 + \frac{1}{4} = \frac{5}{4}$. From there on the breakpoints between subintervals will be every $\frac{1}{4}$, through $x_7 = \frac{11}{4}$ and $x_8 = 3 = \frac{12}{4}$. It will help in the arithmetic if we think of all of those nine points in terms of fourths, as indicated.

(a) Notice that the function $f(x) = \frac{1}{x}$ is decreasing on the interval $[1, 3]$ so by using left endpoints we will be <u>over</u>estimating the integral. The Riemann sum will have eight terms. Each will take the width of a subinterval ($\Delta x = \frac{1}{4}$ in all cases) and multiply by $f(x) = \frac{1}{x}$ evaluated at the left end of that subinterval. Evaluating f at the left ends, $x_0, \ldots, x_7$, gives values $\frac{1}{4/4} = \frac{4}{4}$, $\frac{1}{5/4} = \frac{4}{5}$, $\ldots$, $\frac{1}{11/4} = \frac{4}{11}$. Multiplying by Δx and adding we get

$$\frac{1}{4} \times \frac{4}{4} + \frac{1}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{4}{6} + \cdots + \frac{1}{4} \times \frac{4}{11} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots + \frac{1}{11}$$

$$= \frac{32981}{27720} \approx 1.186544012.$$

(b) Using the trapezoidal rule with $n = 8$ and the points $x_0, \ldots, x_8$ as the ends of the subintervals, we get

$$\frac{1}{4} \times \frac{1}{2} \left(\frac{4}{4} + 2 \times \frac{4}{5} + 2 \times \frac{4}{6} + \cdots + 2 \times \frac{4}{11} + \frac{4}{12} \right)$$

$$= \frac{1}{2} \left(\frac{1}{4} + \frac{2}{5} + \frac{2}{6} + \cdots + \frac{2}{11} + \frac{1}{12} \right) = \frac{1}{2} \left(\frac{30581}{13860} \right) = \frac{30581}{27770}$$

$$\approx 1.103210678.$$

(c) Using Simpson's rule with $n = 8$ and the points $x_0, \ldots, x_8$ as the ends of the subintervals, we get

$$\frac{1}{4} \times \frac{1}{3} \left(\frac{4}{4} + 4 \times \frac{4}{5} + 2 \times \frac{4}{6} + 4 \times \frac{4}{7} + 2 \times \frac{4}{8} + 4 \times \frac{4}{9} + 2 \times \frac{4}{10} + 4 \times \frac{4}{11} + \frac{4}{12} \right)$$

$$= \frac{1}{3} \left(\frac{1}{4} + \frac{4}{5} + \frac{2}{6} + \frac{4}{7} + \frac{2}{8} + \frac{4}{9} + \frac{2}{10} + \frac{4}{11} + \frac{1}{12} \right)$$

$$= \frac{1}{3}\left[\left(\frac{1}{4}+\frac{1}{12}\right)+4\left(\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}\right)+2\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{10}\right)\right]$$

$$= \frac{1}{3}\left(\frac{9137}{2772}\right) = \frac{9137}{8316} \approx 1.098725349.$$

(d) $$\int_1^3 \frac{1}{x}\,dx = \ln|x|\Big]_1^3 = \ln 3 - \ln 1 = \ln 3 \approx 1.098612289.$$