Your Name: ____

Mathematics 222 | **Lecture 1** | **Wilson**

Second Midterm Exam **April 20, 2004**

- **Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **On the other side of this sheet there are some facts and formulas and a table for undetermined coefficients.**
- **Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.**
- **You may refer to notes you have brought in, as announced in class.**

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	12	
2	12	
3	13	
4	13	
5	12	
6	13	
7	13	
8	12	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\,\tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\,du = \frac{1}{n+1}u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\,du = \ln|\sec(u)+\tan(u)| + C$
6. $\int u\,dv = uv - \int v\,du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x\,a^y$
3. $a^b = e^{b\ln a}$

Terms to use in y_p, for undetermined coefficients:

For a term in $f(x)$ which is a multiple of	If	Then use a term like
$\sin(kx)$ or $\cos(kx)$	ki is not a root of the characteristic equation	$A\cos(kx) + B\sin(kx)$
	ki is a root of the characteristic equation	$Ax\cos(kx) + Bx\sin(kx)$
e^{nx}	n is not a root of the characteristic equation	Ce^{nx}
	n is a single root of the characteristic equation	$Cx\,e^{nx}$
	n is a double root of the characteristic equation	$Cx^2\,e^{nx}$
A polynomial ax^2+bx+c of degree at most 2	0 is not a root of the characteristic equation	a polynomial Dx^2+Ex+F of the same degree as ax^2+bx+c
	0 is a single root of the characteristic equation	a polynomial Dx^3+Ex^2+Fx of degree one more
	0 is a double root of the characteristic equation	a polynomial $Dx^4+Ex^3+Fx^2$ of degree two more

Problem 1 (12 points)
A hyperbola crosses the y-axis at $(0, \pm 12)$, and its foci are at $(0, \pm 13)$.

(a) Find an equation for the hyperbola.

(b) What is the eccentricity of this hyperbola?

(c) What are the asymptotes of this hyperbola? (Give equations for the lines.)

Problem 2 (12 points)
Find the interval of convergence (convergence set) for the series

$$\sum_{n=1}^{\infty} \frac{3\,x^n}{n\,2^n}.$$

Be sure to describe the interval fully, including which endpoints (if any) are included in it.

Problem 3 (13 points)
Find the area of the region in the plane which is inside the circle $r = 3\cos\theta$ but outside the cardioid $r = 1 + \cos\theta$.
Be sure to tell how you determine the limits of integration.

Problem 4 (13 points)

Find the Taylor series for $f(x) = \ln(1+3x)$, at $a = \frac{1}{3}$.

You should show the terms through the one with x^4 to get full credit: If in addition you give a correct expression for the n^{th} power term in general you will get extra credit.

Problem 5 (12 points)

(a) Use the Trapezoidal rule with $n = 4$ subintervals to approximate $\int_1^3 (x^2 - 1)\, dx$.

(b) Calculate exactly $\int_1^3 (x^2 - 1)\, dx$.

(c) Give an argument based on the shape of the graph of $y = x^2 - 1$ and the nature of the trapezoidal rule to explain why your answer to (b) is larger or smaller than your answer to (a).

Don't just explain why they are different! Tell why whichever is larger had to be larger.
Hint: think about concavity

Problem 6 (13 points)

We want to use a polynomial consisting of some beginning terms of the Maclaurin series for $\sin(x)$ to approximate $\sin(0.2)$. Use the remainder term $R_n(x)$ from Taylor's theorem to decide which terms to use if the error must be at most $0.000001 = 10^{-6}$.

Write out explicitly what the polynomial should be, i.e. which terms you would use: Don't just tell how many or what degree.

Problem 7 (13 points)
Here is the problem as it appeared on the exam:

Find all solutions of the differential equation $y'' + 6y' - 13y = 9e^{-x}$.

Here is what was intended: The arithmetic is slightly less messy!

Find all solutions of the differential equation $y'' + 6y' + 13y = 16e^{-x}$.

Problem 8 (12 points)
An ellipse is parametrized as $x = 2\sin(t)$ and $y = 4\cos(t)$. Find an equation for the tangent line to this curve at the point where $t = \frac{\pi}{4}$.