Your Name: ____

Mathematics 222 **Lecture 1** **Wilson**

First Midterm Exam **February 24, 2004**

- **Write your answers to the eight problems in the spaces provided. If you must continue an answer somewhere other than immediately after the problem statement, be sure (a) to tell where to look for the answer, and (b) to label the answer wherever it winds up. In any case, be sure to circle your final answer to each problem.**
- **On the other side of this sheet there is a collection of facts and formulas.**
- **Wherever applicable, leave your answers in exact forms (using π, e, $\sqrt{3}$, $\ln(2)$, and similar numbers) rather than using decimal approximations.**
- **You may refer to notes you have brought in on one index card as announced in class.**

BE SURE TO SHOW YOUR WORK, AND EXPLAIN WHAT YOU DID. YOU MAY RECEIVE REDUCED OR ZERO CREDIT FOR UNSUBSTANTIATED ANSWERS. ("I did it on my calculator" and "I used a formula from the book" are not sufficient substantiation...)

Problem	Points	Score
1	14	
2	12	
3	12	
4	10	
5	10	
6	15	
7	15	
8	12	
TOTAL	100	

Some formulas, identities, and numeric values you might find useful:

Values of trig functions:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	—

Trig facts:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
2. $\sec\theta = \frac{1}{\cos\theta}$
3. $\sin^2\theta + \cos^2\theta = 1$
4. $\sec^2\theta = \tan^2\theta + 1$
5. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
6. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
7. $\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
8. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
9. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Derivative formulas:

1. $\frac{d}{dx}\tan x = \sec^2 x$
2. $\frac{d}{dx}\sec x = \sec x\, \tan x$
3. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
5. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
6. $\frac{d}{dx}\ln x = \frac{1}{x}$
7. $\frac{d}{dx}e^x = e^x$

Integral formulas:

1. $\int u^n\, du = \frac{1}{n+1}u^{n+1} + C$, if $n \neq -1$
2. $\int \frac{1}{u}\, du = \ln|u| + C$
3. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$
4. $\int \frac{du}{1+u^2} = \tan^{-1} u + C$
5. $\int \sec(u)\, du = \ln|\sec(u) + \tan(u)| + C$
6. $\int u\, dv = uv - \int v\, du$

Algebra formulas:

1. $\ln(xy) = \ln(x) + \ln(y)$
2. $a^{x+y} = a^x a^y$
3. $a^b = e^{b\ln a}$

Problem 1 (14 points)
Evaluate the integrals:

(a) $$\int \sin^2(x)\ \cos^3(x)\, dx$$

(b) $$\int_0^{\frac{\pi}{6}} \cos^2(x) \sin^2(x)\, dx$$

Problem 2 (12 points)

(a) Evaluate the integral: $$\int \frac{4x^2}{(1-x^2)^{\frac{3}{2}}}\, dx$$

(b) Convert this integral to an integral of a trigonometric function: You do not have to evaluate the resulting integral.

$$\int \frac{dx}{\sqrt{4+x^2}}$$

Problem 3 (12 points)
Evaluate the integrals:

(a) $$\int e^x \, (x^2 - 5x) \, dx$$

(b) $$\int \arctan(x) \, dx$$

Problem 4 (10 points)
Evaluate the integral:

$$\int_0^3 \frac{dx}{(x-1)^{\frac{2}{5}}}$$

Problem 5 (10 points)
Evaluate the integrals:

(a) $$\int_1^\infty x\, e^{-x}\, dx$$

(b) $$\int_0^\infty \frac{1}{\sqrt{x+2}}\, dx$$

Problem 6 (15 points)

For each series, tell whether it converges or diverges and give a reason for your answer.

(a) $$\sum_{n=1}^{\infty} \frac{n^2+3n+2}{2n^2-1}$$

(b) $$\sum_{n=2}^{\infty} e^{-n}$$

(c) $$\sum_{n=1}^{\infty} \frac{3^n+1000}{6+7^n}$$

Problem 7 (15 points)

For each series, tell whether it converges absolutely, converges conditionally, or does not converge at all, and give a reason for your answer.

(a) $$\sum_{n=1}^{\infty} \frac{1}{(-4)^n}$$

(b) $$\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{100n} \right)$$

(c) $$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$$

Problem 8 (12 points)

Find the sum of the series $$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}.$$

(Your answer should not be just a number that might have come from a calculator, but reasoned steps leading to a numeric answer.)

Hints:

(i) Find numbers A and B such that $\frac{2}{n^2+4n+3} = \frac{A}{n+1} + \frac{B}{n+3}$.

(ii) Find an expression for the general n^{th} partial sum of the series, making use of the A and B you found in (i).

(iii) Use the definition of the sum of an infinite series.