Course: Math 535 - Mathematical Methods in Data Science (MMiDS) - Spring 2024

HWK 6

1 Establish that $U^\perp$ is a linear subspace.

2 Let $A \in \mathbb{R}^{m \times r}$ and $B \in \mathbb{R}^{r \times n}$.

a) Show that $\mathrm{rk}(A^T) = \mathrm{rk}(A)$.

b) Deduce that $\mathrm{rk}(AB) \leq \mathrm{rk}(B)$.

3 Let $A \in \mathbb{R}^{m \times r}$ and $B \in \mathbb{R}^{r \times n}$, both of rank r.

a) Show that $A^T A$, BB^T and $A^T ABB^T$ are all invertible.

b) Show that $r = \mathrm{rk}(A^T ABB^T) \leq \mathrm{rk}(AB)$.

c) Conclude that $\mathrm{rk}(AB) = r$.

4 Construct a matrix $A \in \mathbb{R}^{n \times n}$ for which there exist multiple solutions to the maximization problem

$$\mathbf{v}_1 \in \arg\max\{\|A\mathbf{v}\| : \|\mathbf{v}\| = 1\}.$$