Chapter 10
The Manipulability of Voting Systems

Chapter Outline

Chapter Summary

Manipulation of a voting system allows a voter or a group of voters to change the outcome of an election based on the knowledge of the other voters' preference list ballots. The different voting systems examined in Chapter 9 will again be considered in light of whether they can be manipulated or not. A voting system is said to be *manipulable* by a *unilateral* change if there exist two sequences of preference list ballots and a voter such that neither election results in a tie, the only ballot change is by voter casting the *disingenuous ballot,* and that voter prefers the outcome of the second election compared to the first.

Majority rule and Condorcet's Method will be shown to be non-manipulable given certain conditions, which includes an odd number of voters. Borda count will be manipulable under certain conditions and non-manipulable given others. The same will be shown for plurality voting in which group manipulation is the condition for manipulation. The runoff systems discussed in Chapter 9 are revisited. Both plurality runoff and the Hare system turn out to be manipulable. Finally, sequential pairwise voting will be examined in two ways. This voting system can be manipulated by a unilateral change and a fixed agenda. This voting system can also be manipulated not by altering a preference list ballot, but by altering the agenda.

May's theorem for manipulability establishes that given the initial conditions (odd number of voters and only two candidates) majority rule is the only voting method that satisfies three important properties. This theorem recognizes that majority rule is non-manipulable. Another important theorem discussed in this chapter is the *Gibbard-Satterthwaite Theorem* which says that with three or more candidates and any number of voters, there does not exist (and never will exist) a voting system that always has a winner, no ties, satisfies the Pareto condition, is non-manipulable, and is not a dictatorship.

Finally, this chapter will examine the *chair's paradox*. The interesting outcome involved in the chair's paradox is that although the chair has tie-breaking power, he or she would be in a better strategic advantage by handing this power off to another voter.

Skill Objectives

1. Explain what is meant by voting manipulation.

2. Determine if a voter, by a unilateral change, has manipulated the outcome of an election.

3. Determine a unilateral change by a voter that causes manipulation of an election with different in the Borda count voting method.

4. Explain the three conditions to determine if a voting system is manipulable.

5. Discuss why the majority method may not be appropriate for an election in which there are more than two candidates.

6. Explain four desirable properties of Condorcet's method.

7. Explain why Condorcet's method is non-manipulable by a unilateral change in vote.

8. Recognize when the Borda count method can be manipulated and when it can't.

9. Determine a unilateral change by a voter that causes a no-winner manipulation of an election in Condorcet's method.

10. Determine a unilateral change by a voter that causes manipulation of an election in the plurality runoff method.

11. Determine a unilateral change by a voter that causes manipulation of an election in the Borda count voting method.

12. Determine a unilateral change by a voter that causes manipulation of an election in the Hare method.

13. Determine a group change by a block of voters that causes manipulation of an election in the plurality method.

14. Determine an agenda change by a voter that causes manipulation of an election in the sequential pairwise voting method, with agenda.

15. Explain the Gibbard-Satterthwaite theorem (GS theorem) and its weak version.

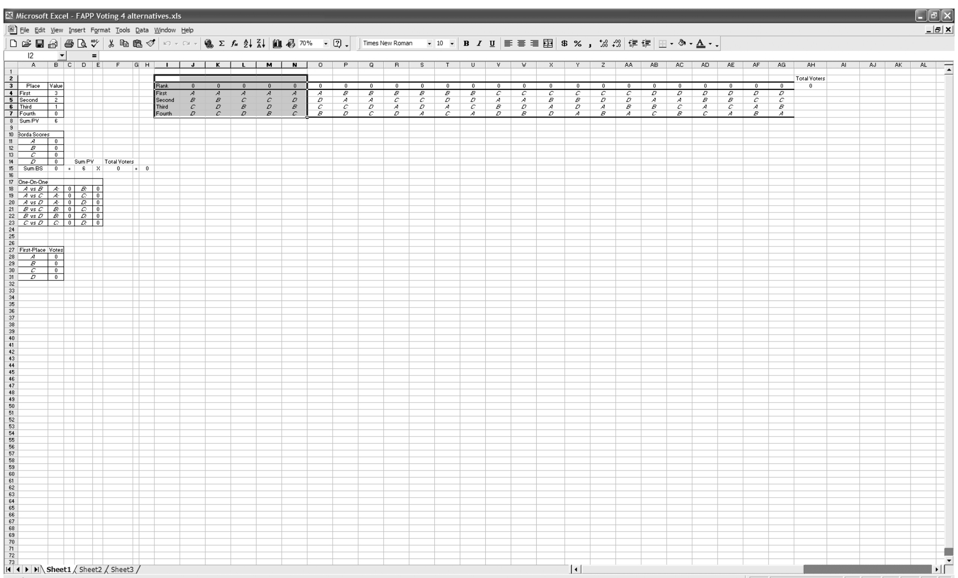
16. Explain the chair's paradox and what is meant by *weakly dominates* as it relates to a voting strategy.

Teaching Tips

1. In trying to determine what unilateral change will cause a manipulation to an election, you may choose to point out to students that they must always first determine the outcome using the original preference list ballots. Given voters that will cast the disingenuous ballot and their true preferences, focus on the candidates above the winner of Election 1. If any of these can become the winner of Election 2, then voters have manipulated the outcome by choosing a more-preferred candidate. It is possible that candidate may not be their first choice.

2. Students may need a quick review of the voting method used in the discussion of each (non) manipulation. By briefly describing the method, you may find it easier to discuss whether the system is manipulable or not. Also, state the conditions such as whether or not only an odd number of voters is considered.

3. You should find the Excel spreadsheets for 3 and 4 candidates a tremendous help in either determining how to manipulate the outcome of an election or by showing that a particular voter cannot get a more-preferred outcome. They are particularly helpful in demonstrating manipulation in the Borda count voting system, sequential pairwise voting, as well as sequential pairwise voting, with agenda. Suppose you want to make a handout that has four candidates and five voters. To get a template for this, you can copy part of the spreadsheet and paste it into a word processor such as Word.



Rank	0	0	0	0	0
First	*A*	*A*	*A*	*A*	*A*
Second	*B*	*B*	*C*	*C*	*D*
Third	*C*	*D*	*B*	*D*	*B*
Fourth	*D*	*C*	*D*	*B*	*C*

You can then make you choices of preference list ballots in the spreadsheet. If you are using the Borda Count, the results are calculated given the place values you choose. The default is 0, 1, 2, and 3 for the four-candidate spreadsheet. You can copy and the paste the results in the document.

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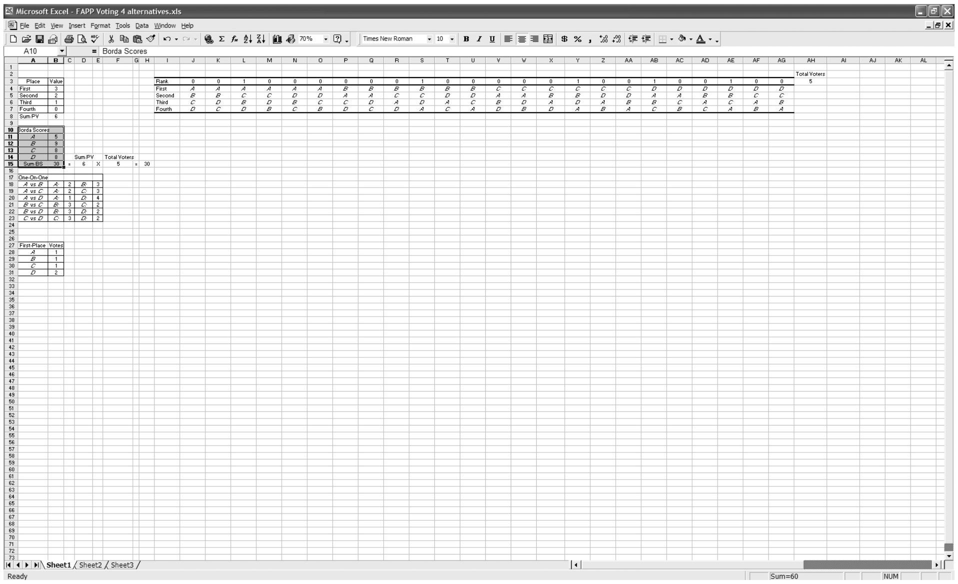
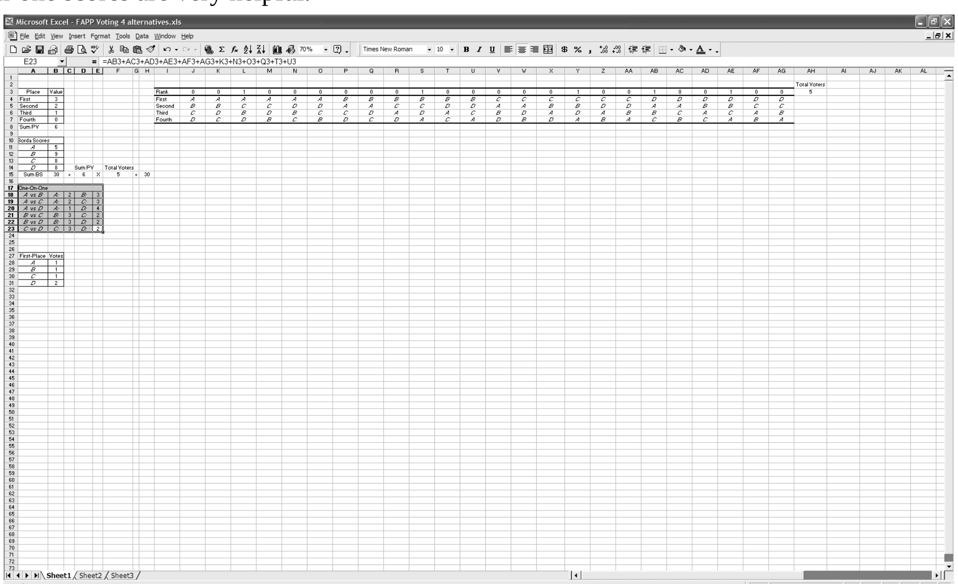


Borda Scores	
A	5
B	9
C	8
D	8
Sum BS	30

Similarly if you are using sequential pairwise voting and wish to manipulate the agenda, the one-on-one scores are very helpful.



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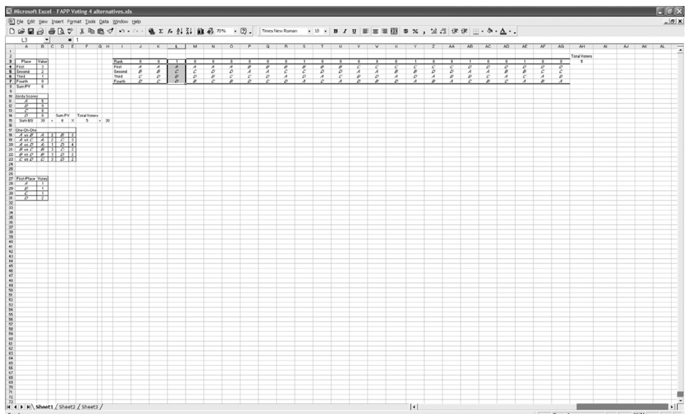
One-On-One				
A vs *B*	*A:*	2	*B:*	3
A vs *C*	*A:*	2	*C:*	3
A vs *D*	*A:*	1	*D:*	4
B vs *C*	*B:*	3	*C:*	2
B vs *D*	*B:*	3	*D:*	2
C vs *D*	*C:*	3	*D:*	2

To complete the desired preference list ballots, you can individually copy and paste them into your template.



Rank	1	1	1	1	1
First	*A*	*B*	*C*	*D*	*D*
Second	*C*	*C*	*B*	*A*	*B*
Third	*B*	*D*	*D*	*B*	*C*
Fourth	*D*	*A*	*A*	*C*	*A*

4. In discussing a system that is manipulable, don't be surprised if students question whether it really is manipulable by giving what they feel is a counter-example. In discussing manipulable systems, you may wish to make the point more than once that a system is manipulable if there is at least one scenario in which manipulation occurs. Because of the possibility of ties with an even number of voters, students may come to the conclusion that a system is not manipulable because they may not be able to manipulate certain examples.

5. Students may view voting manipulation as kind of a "do over" so to speak. You may wish to convey to students that Election 2 is the election that prevails due to the casting of the disingenuous ballot.

Research Paper

Have students investigate the life and contributions of Ramon Llull (1235–1316), the Spanish mathematician and missionary. Llull proposed in the 13th century a voting system based on the principle of fairness. Other figures such as Nicholas of Cusa (1401–1464) had access to some works by Llull. However, a full description of Llull's voting system was not published until 2001! Some of Cusa's work centered on how to elect German kings while Llull's work had an impact 500 years later on the work of Condorcet.

Collaborative Learning

Rock Paper Scissors

Variations of the game Rock Paper Scissors (RPS) have been around for a very long time. Even as early as 50,000 B.C. hand games had been used to resolve issues such as mating and food issues. The first variation of Rock Paper Scissors was known as Janken. The earliest possible date for the actual name of Rock Paper Scissors would be around 500 A.D., when scissors were invented by a hair cutter (Isidore of Seville). This game has made headlines recently when two action houses (Sotheby's and Christie's) played RPS to settle who would receive the rights to sell four multimillion-dollar paintings.

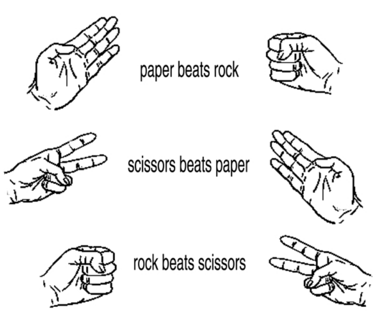
Although there are different variations of RPS, in the case of three options, the principle is the same. The system is known as *non-transitive*. If a relation is transitive then if A is preferred to B and B is preferred to C, then it is necessary that A is preferred to C. In the case of a non-transitive relation then a loop of preference occurs. For example:

- A is preferred to B
- B is preferred to C
- C is preferred to A

A set-up like this is reminiscent of the Condorcet's voting paradox of Chapter 9 where three or more candidates in an election yield no winner using Condorcet's method.

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*A*	*B*	*C*
Second	*B*	*C*	*A*
Third	*C*	*A*	*B*

In the game, we have the following rules with the hand symbols.



Now this game is generally played with two players only. But let's assume be have three players and they rank their choices

Game 1

	Number of players (3)		
Round	**YOU**	**Player *X***	**Player *Y***
First	*Rock*	*Rock*	*Paper*
Second	*Paper*	*Scissors*	*Scissors*
Third	*Scissors*	*Paper*	*Rock*

In the first round, Player Y beats You and Player X. Player Y is declared the winner. Suppose you knew how Player X and Player Y were going to make their preferences. You could alter your choices.

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Game 2

Round	Number of players (3)		
	YOU	**Player *X***	**Player *Y***
First	*Paper*	*Rock*	*Paper*
Second	*Rock*	*Scissors*	*Scissors*
Third	*Scissors*	*Paper*	*Rock*

In the first round, You and Player *Y* beat Player *X*. You and Player *Y* go to the second round. Since rock beats scissors, you are declared the winner.

You were able to manipulate the outcome of the game by knowing the choices that would be made by Players *X* and *Y*.

In the following tables, determine what the outcome of the contest would be if your actual preference list is as follows.

YOU
Rock
Paper
Scissors

Then try to find a sequence of *Rock Paper Scissors* that would manipulate the outcome. If you were the winner, then no manipulation is necessary. Note that if *Rock Paper Scissors* all appear in the first round, then there is a three-way tie and all three players go forward to the second round. With three players in the second round, if there is a three-way tie, then the game would be tied.

Round	Number of players (3)		
	YOU	**Player *X***	**Player *Y***
First		*Rock*	*Paper*
Second		*Scissors*	*Rock*
Third		*Paper*	*Scissors*

Round	Number of players (3)		
	YOU	**Player *X***	**Player *Y***
First		*Rock*	*Rock*
Second		*Scissors*	*Scissors*
Third		*Paper*	*Paper*

Round	Number of players (3)		
	YOU	**Player *X***	**Player *Y***
First		*Rock*	*Rock*
Second		*Paper*	*Paper*
Third		*Scissors*	*Scissors*

Round	Number of players (3)		
	YOU	**Player *X***	**Player *Y***
First		*Paper*	*Scissors*
Second		*Scissors*	*Paper*
Third		*Rock*	*Rock*

Now, have two players (*X* and *Y*) actually play Rock Paper Scissors and record the results. In this case any of the three options can appear more than once.

Game 1

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 2

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 3

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 4

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 5

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 6

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

Game 7

	Number of players (3)		
Round	**Player *Z***	**Player *X***	**Player *Y***
First			
Second			
Third			

For each game involving Players *X* and *Y*, state who the winner is using the rule that 2 out of 3 or 3 out of 3 winning rounds win a game. It is possible to have a two-way tie of a game.

After completing all seven games, determine if the third player (Player *Z*) can manipulate the outcome if he or she must use all three choices of Rock, Paper, and Scissors. The manipulation would be done in the same way as it was shown at first, round by round.

Now consider a variation of the RPS where you add *Water* and *Dynamite*. Dynamite blows up rock and burns paper. Dynamite is put out by water and the fuse is cut by scissors. Water puts out dynamite and rusts scissors. Water is splashed by rock and absorbed by paper.

Have each player independently create their preference list ballot (no repeats) and put them in the table. Determine the outcome of the rounds.

	Number of players (3)		
Round	**Player *A***	**Player *B***	**Player *C***
First			
Second			
Third			
Fourth			
Fifth			

Have each player see if they can manipulate the outcome by changing their preferences.

Now have all three players play games by using the symbol of pointing a finger for dynamite ☜ and a spread out hand for water 🖐.

For each round, discuss how a player could alter the results by making a different choice. Do this five times.

Game 1

Number of players (3)		
Player *A*	**Player *B***	**Player *C***

Game 2

Number of players (3)		
Player *A*	**Player *B***	**Player *C***

Game 3

Number of players (3)		
Player *A*	**Player *B***	**Player *C***

Game 4

Number of players (3)		
Player *A*	**Player *B***	**Player *C***

Game 5

Number of players (3)		
Player *A*	**Player *B***	**Player *C***

Solutions

Skills Check:

1. c	2. c	3. c	4. c	5. a	6. c	7. c	8. b	9. a	10. b
11. c	12. a	13. c	14. c	15. a	16. a	17. d	18. b	19. a	20. b

1. One example of two such elections is the following:

Election 1

Rank	**Number of voters (3)**		
First	*A*	*A*	*B*
Second	*B*	*B*	*A*

Election 2

Rank	**Number of voters (3)**		
First	*B*	*A*	*B*
Second	*A*	*B*	*A*

With the voting system in which the candidate with the fewest first-place votes wins, *B* is the winner in the first election. However, if the leftmost voter changes his or her ballot as shown in the second election, then *A* becomes the winner. Taking the ballots in the first election to be the sincere preferences of the voters, the leftmost voter (who prefers *A* to *B*) has secured a more favorable outcome by the submission of a disingenuous ballot.

2. One example of two such elections is the following:

Election 1

Rank	**Number of voters (3)**		
First	*A*	*A*	*B*
Second	*B*	*B*	*A*

Election 2

Rank	**Number of voters (3)**		
First	*B*	*A*	*B*
Second	*A*	*B*	*A*

With the voting system in which the candidate receiving an odd number of first-place votes wins, *B* is the winner in the first election. However, if the leftmost voter changes his or her ballot as shown in the second election, then *A* becomes the winner. Taking the ballots in the first election to be the sincere preferences of the voters, the leftmost voter (who prefers *A* to *B*) has secured a more favorable outcome by the submission of a disingenuous ballot.

3. One example of two such elections is the following:

Election 1

Rank	**Number of voters (3)**		
First	*A*	*B*	*B*
Second	*B*	*A*	*A*

Election 2

Rank	**Number of voters (3)**		
First	*B*	*B*	*B*
Second	*A*	*A*	*A*

With the voting system in which the candidate receiving an even number of first-place votes wins, *B* is the winner in the first election. However, if the leftmost voter changes his or her ballot as shown in the second election, then *A* becomes the winner. Taking the ballots in the first election to be the sincere preferences of the voters, the leftmost voter (who prefers *A* to *B*) has secured a more favorable outcome by the submission of a disingenuous ballot.

4. (a) The voting system does not treat both *candidates* the same.

(b) Candidate *A* wins regardless of the ballots.

(c) Candidate *B* wins regardless of the ballots.

5. (a) The voting system does not treat all *voters* the same.

(b) A dictatorship in which Voter #1 is the dictator.

(c) A dictatorship in which Voter #2 is the dictator and a dictatorship in which voter #3 is the dictator.

6.

22%	**23%**	**15%**	**29%**	**7%**	**4%**
D	*D*	*H*	*H*	*J*	*J*
H	*J*	*D*	*J*	*H*	*D*
J	*H*	*J*	*D*	*D*	*H*

We must check the one-on-one scores of *D* versus *H*, *D* versus *J*, and *H* versus *J*.

D versus *H*:*D* is over *H* on 22% + 23% + 4% = 49% of the ballots, while the reverse is true on 15% + 29% + 7% = 51%. Thus, *H* defeats *D*, 51% to 49%.

D versus *J*: *D* is over *J* on 22% + 23% + 15% = 60% of the ballots, while the reverse is true on 29% + 7% + 4% = 40%. Thus, *D* defeats *J*, 60% to 40%.

H versus *J*: *H* is over *J* on 22% + 15% + 29% = 66% of the ballots, while the reverse is true on 23% + 7% + 4% = 34%. Thus, *H* defeats *J*, 66% to 34%.

Since *H* can defeat both *D* and *J* in a one-to one competition, Elizabeth Holtzman (*H*) is the winner by Condorcet's method.

7. Election 1

	Number of voters (2)	
Rank	**1**	**1**
First	*B*	*A*
Second	*C*	*D*
Third	*A*	*C*
Fourth	*D*	*B*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	1×3	0×2	1×1	0×0	4
B	1×3	0×2	0×1	1×0	3
C	0×3	1×2	1×1	0×0	3
D	0×3	1×2	0×1	1×0	2

With the given ballots, the winner using the Borda count is *A*. However, if the leftmost voter changes his or her preference ballot, we have the following.

Election 2

	Number of voters (2)	
Rank	**1**	**1**
First	*C*	*A*
Second	*B*	*D*
Third	*D*	*C*
Fourth	*A*	*B*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	1×3	0×2	0×1	1×0	3
B	0×3	1×2	0×1	1×0	2
C	1×3	0×2	1×1	0×0	4
D	0×3	1×2	1×1	0×0	3

With the new ballots, the winner using the Borda count is *C*.

8. One way to get an example of manipulation of the Borda count with seven candidates and eight voters is to alter the elections in Example 2 of the text by adding *F* and *G* to the bottom of each of the six ballots in both elections, and then adding the two rightmost columns as shown. The last two voters contribute exactly 6 to the Borda score of each candidate, and so, taken together have no effect on who is the winner of the election.

Election 1

	Number of voters (8)							
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*A*	*E*	*A*	*E*	*A*	*G*
Second	*B*	*C*	*B*	*D*	*B*	*D*	*B*	*F*
Third	*C*	*A*	*C*	*C*	*C*	*C*	*C*	*E*
Fourth	*D*	*D*	*D*	*B*	*D*	*B*	*D*	*D*
Fifth	*E*	*E*	*E*	*A*	*E*	*A*	*E*	*C*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*B*
Seventh	*G*	*G*	*G*	*G*	*G*	*G*	*G*	*A*

Preference	1st place votes × 6	2nd place votes × 5	3rd place votes × 4	4th place votes × 3	5th place votes × 2	6th place votes × 1	7th place votes × 0	Borda score
A	4×6	0×5	1×4	0×3	2×2	0×1	1×0	32
B	1×6	4×5	0×4	2×3	0×2	1×1	0×0	33
C	0×6	1×5	6×4	0×3	1×2	0×1	0×0	31
D	0×6	2×5	0×4	6×3	0×2	0×1	0×0	28
E	2×6	0×5	1×4	0×3	5×2	0×1	0×0	26
F	0×6	1×5	0×4	0×3	0×2	7×1	0×0	12
G	1×6	0×5	0×4	0×3	0×2	0×1	7×0	6

Thus, *B* has the highest Borda score and is declared the winner. This was the expected result.

The voter on the far left prefers *A* to *B*. By casting a disingenuous ballot (still preferring *A* to *B* though), the outcome of the election is altered.

Election 2

	Number of voters (8)							
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*A*	*E*	*A*	*E*	*A*	*G*
Second	*D*	*C*	*B*	*D*	*B*	*D*	*B*	*F*
Third	*C*	*A*	*C*	*C*	*C*	*C*	*C*	*E*
Fourth	*B*	*D*	*D*	*B*	*D*	*B*	*D*	*D*
Fifth	*E*	*E*	*E*	*A*	*E*	*A*	*E*	*C*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*B*
Seventh	*G*	*G*	*G*	*G*	*G*	*G*	*G*	*A*

Preference	1st place votes × 6	2nd place votes × 5	3rd place votes × 4	4th place votes × 3	5th place votes × 2	6th place votes × 1	7th place votes × 0	Borda score
A	4×6	0×5	1×4	0×3	2×2	0×1	1×0	32
B	1×6	3×5	0×4	3×3	0×2	1×1	0×0	31
C	0×6	1×5	6×4	0×3	1×2	0×1	0×0	31
D	0×6	3×5	0×4	5×3	0×2	0×1	0×0	30
E	2×6	0×5	1×4	0×3	5×2	0×1	0×0	26
F	0×6	1×5	0×4	0×3	0×2	7×1	0×0	12
G	1×6	0×5	0×4	0×3	0×2	0×1	7×0	6

Thus, *A* has the highest Borda score and is declared the winner.

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8. continued

One could also add two ballots canceling each other out first, and then add *F* and *G* to the bottom of all eight ballots in each election. By doing this, the last two voters contribute exactly 6 to the Borda score of each of the top five candidates, and so taken together have no effect on who is the winner of the election. Because *F* and *G* hold the sixth and seventh places, respectively, on all ballots, they have no effect on the candidates above them.

Election 1

	Number of voters (8)							
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*A*	*E*	*A*	*E*	*A*	*E*
Second	*B*	*C*	*B*	*D*	*B*	*D*	*B*	*D*
Third	*C*	*A*	*C*	*C*	*C*	*C*	*C*	*C*
Fourth	*D*	*D*	*D*	*B*	*D*	*B*	*D*	*B*
Fifth	*E*	*E*	*E*	*A*	*E*	*A*	*E*	*A*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
Seventh	*G*	*G*	*G*	*G*	*G*	*G*	*G*	*G*

Preference	1st place votes × 6	2nd place votes × 5	3rd place votes × 4	4th place votes × 3	5th place votes × 2	6th place votes × 1	7th place votes × 0	Borda score
A	4×6	0×5	1×4	0×3	3×2	0×1	0×0	34
B	1×6	4×5	0×4	3×3	0×2	0×1	0×0	35
C	0×6	1×5	7×4	0×3	0×2	0×1	0×0	33
D	0×6	3×5	0×4	5×3	0×2	0×1	0×0	30
E	3×6	0×5	0×4	0×3	5×2	0×1	0×0	28
F	0×6	0×5	0×4	0×3	0×2	8×1	0×0	8
G	0×6	0×5	0×4	0×3	0×2	0×1	8×0	0

Thus, *B* has the highest Borda score and is declared the winner. This was the expected result.

The voter on the far left prefers *A* to *B*. By casting a disingenuous ballot (still preferring *A* to *B* though), the outcome of the election is altered.

Election 2

	Number of voters (8)							
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*A*	*E*	*A*	*E*	*A*	*E*
Second	*D*	*C*	*B*	*D*	*B*	*D*	*B*	*D*
Third	*C*	*A*	*C*	*C*	*C*	*C*	*C*	*C*
Fourth	*B*	*D*	*D*	*B*	*D*	*B*	*D*	*B*
Fifth	*E*	*E*	*E*	*A*	*E*	*A*	*E*	*A*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*F*	*F*
Seventh	*G*	*G*	*G*	*G*	*G*	*G*	*G*	*G*

Preference	1st place votes × 6	2nd place votes × 5	3rd place votes × 4	4th place votes × 3	5th place votes × 2	6th place votes × 1	7th place votes × 0	Borda score
A	4×6	0×5	1×4	0×3	3×2	0×1	0×0	34
B	1×6	3×5	0×4	4×3	0×2	0×1	0×0	33
C	0×6	1×5	7×4	0×3	0×2	0×1	0×0	33
D	0×6	4×5	0×4	4×3	0×2	0×1	0×0	32
E	3×6	0×5	0×4	0×3	5×2	0×1	0×0	28
F	0×6	0×5	0×4	0×3	0×2	8×1	0×0	8
G	0×6	0×5	0×4	0×3	0×2	0×1	8×0	0

Thus, *A* has the highest Borda score and is declared the winner.

9. Election 1

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*A*	*B*	*B*
Second	*B*	*A*	*A*
Third	*C*	*C*	*C*
Fourth	*D*	*D*	*D*

With the given ballots, the winner using the Borda count is *B*.

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	1×3	2×2	0×1	0×0	7
B	2×3	1×2	0×1	0×0	8
C	0×3	0×2	3×1	0×0	3
D	0×3	0×2	0×1	3×0	0

The voter on the far left prefers *A* to *B*. By casting a disingenuous ballot (still preferring *A* to *B* though), the outcome of the election is altered.

Election 2

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*A*	*B*	*B*
Second	*C*	*A*	*A*
Third	*D*	*C*	*C*
Fourth	*B*	*D*	*D*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	1×3	2×2	0×1	0×0	7
B	2×3	0×2	0×1	1×0	6
C	0×3	1×2	2×1	0×0	4
D	0×3	0×2	1×1	2×0	1

Thus, *A* has the highest Borda score and is declared the winner.

10. The last two voters contribute exactly 3 to the Borda score of each candidate, and so taken together have no effect on who is the winner of the election. The desired ballots (obtained as suggested in the statement of the exercise) are as follows.

Election 1

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*D*
Second	*B*	*A*	*A*	*B*	*C*
Third	*C*	*C*	*C*	*C*	*B*
Fourth	*D*	*D*	*D*	*D*	*A*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	2×3	2×2	0×1	1×0	10
B	2×3	2×2	1×1	0×0	11
C	0×3	1×2	4×1	0×0	6
D	1×3	0×2	0×1	4×0	3

Thus, *B* has the highest Borda score and is declared the winner. This was the expected result.

Continued on next page

10. continued

The voter on the far left prefers *A* to *B*. By casting a disingenuous ballot (still preferring *A* to *B* though), the outcome of the election is altered.

Election 2

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*D*
Second	*D*	*A*	*A*	*B*	*C*
Third	*C*	*C*	*C*	*C*	*B*
Fourth	*B*	*D*	*D*	*D*	*A*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	2×3	2×2	0×1	1×0	10
B	2×3	1×2	1×1	1×0	9
C	0×3	1×2	4×1	0×0	6
D	1×3	1×2	0×1	3×0	5

Thus, *A* has the highest Borda score and is declared the winner.

11. The following is one such example:

Election 1

	Number of voters (9)								
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*D*	*A*	*F*	*A*	*F*
Second	*B*	*A*	*A*	*B*	*C*	*B*	*E*	*B*	*E*
Third	*C*	*C*	*C*	*C*	*B*	*C*	*D*	*C*	*D*
Fourth	*D*	*D*	*D*	*D*	*A*	*D*	*C*	*D*	*C*
Fifth	*E*	*E*	*E*	*E*	*E*	*E*	*B*	*E*	*B*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*A*	*F*	*A*

Preference	1st place votes × 5	2nd place votes × 4	3rd place votes × 3	4th place votes × 2	5th place votes × 1	6th place votes × 0	Borda score
A	4×5	2×4	0×3	1×2	0×1	2×0	30
B	2×5	4×4	1×3	0×2	2×1	0×0	31
C	0×5	1×4	6×3	2×2	0×1	0×0	26
D	1×5	0×4	2×3	6×2	0×1	0×0	23
E	0×5	2×4	0×3	0×2	7×1	0×0	15
F	2×5	0×4	0×3	0×2	0×1	7×0	10

Thus, *B* has the highest Borda score and is declared the winner. This was the expected result.

The voter on the far left prefers *A* to *B*. By casting a disingenuous ballot (still preferring *A* to *B* though), the outcome of the election is altered.

Election 2

	Number of voters (9)								
Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*D*	*A*	*F*	*A*	*F*
Second	*D*	*A*	*A*	*B*	*C*	*B*	*E*	*B*	*E*
Third	*C*	*C*	*C*	*C*	*B*	*C*	*D*	*C*	*D*
Fourth	*B*	*D*	*D*	*D*	*A*	*D*	*C*	*D*	*C*
Fifth	*E*	*E*	*E*	*E*	*E*	*E*	*B*	*E*	*B*
Sixth	*F*	*F*	*F*	*F*	*F*	*F*	*A*	*F*	*A*

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11. continued

Preference	1st place votes × 5	2nd place votes × 4	3rd place votes × 3	4th place votes × 2	5th place votes × 1	6th place votes × 0	Borda score
A	4×5	2×4	0×3	1×2	0×1	2×0	30
B	2×5	3×4	1×3	1×2	2×1	0×0	29
C	0×5	1×4	6×3	2×2	0×1	0×0	26
D	1×5	1×4	2×3	5×2	0×1	0×0	25
E	0×5	2×4	0×3	0×2	7×1	0×0	15
F	2×5	0×4	0×3	0×2	0×1	7×0	10

Thus, *A* has the highest Borda score and is declared the winner.

12. Election 1

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*B*	*D*	*C*	*B*
Second	*C*	*C*	*A*	*A*
Third	*D*	*A*	*B*	*C*
Fourth	*A*	*B*	*D*	*D*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	0×3	2×2	1×1	1×0	5
B	2×3	0×2	1×1	1×0	7
C	1×3	2×2	1×1	0×0	8
D	1×3	0×2	1×1	2×0	4

Thus, *C* has the highest Borda score and is declared the winner. But the winner becomes *B* if the leftmost voter changes his or her ballot as follows.

Election 2

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*B*	*D*	*C*	*B*
Second	*D*	*C*	*A*	*A*
Third	*A*	*A*	*B*	*C*
Fourth	*C*	*B*	*D*	*D*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	0×3	2×2	2×1	0×0	6
B	2×3	0×2	1×1	1×0	7
C	1×3	1×2	1×1	1×0	6
D	1×3	1×2	0×1	2×0	5

Thus, *B* has the highest Borda score and is declared the winner.

13. Election 1

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*A*	*C*	*B*	*D*
Second	*B*	*A*	*D*	*C*
Third	*C*	*B*	*C*	*A*
Fourth	*D*	*D*	*A*	*B*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	1×3	1×2	1×1	1×0	6
B	1×3	1×2	1×1	1×0	6
C	1×3	1×2	2×1	0×0	7
D	1×3	1×2	0×1	2×0	5

Thus, *C* has the highest Borda score and is declared the winner. But the winner becomes *B* if the leftmost voter changes his or her ballot as follows.

Election 2

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*B*	*C*	*B*	*D*
Second	*A*	*A*	*D*	*C*
Third	*D*	*B*	*C*	*A*
Fourth	*C*	*D*	*A*	*B*

Preference	1st place votes × 3	2nd place votes × 2	3rd place votes × 1	4th place votes × 0	Borda score
A	0×3	2×2	1×1	1×0	5
B	2×3	0×2	1×1	1×0	7
C	1×3	1×2	1×1	1×0	6
D	1×3	1×2	1×1	1×0	6

Thus, *B* has the highest Borda score and is declared the winner.

14. Election 1

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*A*	*C*	*A*	*D*
Second	*B*	*E*	*E*	*B*
Third	*C*	*D*	*D*	*E*
Fourth	*D*	*B*	*C*	*C*
Fifth	*E*	*A*	*B*	*A*

We need to determine the winner of 10 one-to-one preferences (ties are possible since we have an even number of voters).

There are 2 voters who prefer *A* to *B* and 2 prefer *B* to *A*. Thus, *A* and *B* tie by a score of 2 to 2. Award *A* and *B* each ½ point.

There are 2 voters who prefer *A* to *C* and 2 prefer *C* to *A*. Thus, *A* and *C* tie by a score of 2 to 2. Award *A* and *C* each ½ point.

There are 2 voters who prefer *A* to *D* and 2 prefer *D* to *A*. Thus, *A* and *D* tie by a score of 2 to 2. Award *A* and *D* each ½ point.

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14. continued

There are 2 voters who prefer *A* to *E* and 2 prefer *E* to *A*. Thus, *A* and *E* tie by a score of 2 to 2. Award *A* and *E* each ½ point.

There are 2 voters who prefer *B* to *C* and 2 prefer *C* to *B*. Thus, *B* and *C* tie by a score of 2 to 2. Award *B* and *C* each ½ point.

There is 1 voter who prefers *B* to *D* and 3 prefer *D* to *B*. Thus, *D* wins by a score of 3 to 1. Award *D* 1 point.

There are 2 voters who prefer *B* to *E* and 2 prefer *E* to *B*. Thus, *B* and *E* tie by a score of 2 to 2. Award *B* and *E* each ½ point.

There are 2 voters who prefer *C* to *D* and 2 prefer *D* to *C*. Thus, *C* and *D* tie by a score of 2 to 2. Award *D* and *C* each ½ point.

There are 2 voters who prefer *C* to *E* and 2 prefer *E* to *C*. Thus, *C* and *E* tie by a score of 2 to 2. Award *C* and *E* each ½ point.

There are 2 voters who prefer *D* to *E* and 2 prefer *E* to *D*. Thus, *D* and *E* tie by a score of 2 to 2. Award *E* and *D* each ½ point.

	A	*B*	*C*	*D*	*E*
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Total	2	$1\frac{1}{2}$	2	$2\frac{1}{2}$	2

With the given ballots, the winner using Copeland's rule is D. But the winner becomes *C* if the leftmost voter changes his or her ballot as the following shows.

Election 2

	Number of voters (4)			
Rank	**1**	**1**	**1**	**1**
First	*C*	*C*	*A*	*D*
Second	*A*	*E*	*E*	*B*
Third	*B*	*D*	*D*	*E*
Fourth	*D*	*B*	*C*	*C*
Fifth	*E*	*A*	*B*	*A*

There are 2 voters who prefer *A* to *B* and 2 prefer *B* to *A*. Thus, *A* and *B* tie by a score of 2 to 2. Award *A* and *B* each ½ point.

There is 1 voter who prefers *A* to *C* and 3 prefer *C* to *A*. Thus, *C* wins by a score of 3 to 1. Award *C* 1 point.

There are 2 voters who prefer *A* to *D* and 2 prefer *D* to *A*. Thus, *A* and *D* tie by a score of 2 to 2. Award *A* and *D* each ½ point.

There are 2 voters who prefer *A* to *E* and 2 prefer *E* to *A*. Thus, *A* and *E* tie by a score of 2 to 2. Award *A* and *E* each ½ point.

There is 1 voter who prefers *B* to *C* and 3 prefer *C* to *B*. Thus, *C* wins by a score of 3 to 1. Award *C* 1 point.

There is 1 voter who prefers *B* to *D* and 3 prefer *D* to *B*. Thus, *D* wins by a score of 3 to 1. Award *D* 1 point.

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14. continued

There are 2 voters who prefer *B* to *E* and 2 prefer *E* to *B*. Thus, *B* and *E* tie by a score of 2 to 2. Award *B* and *E* each ½ point.

There are 2 voters who prefer *C* to *D* and 2 prefer *D* to *C*. Thus, *C* and *D* tie by a score of 2 to 2. Award *D* and *C* each ½ point.

There are 2 voters who prefer *C* to *E* and 2 prefer *E* to *C*. Thus, *C* and *E* tie by a score of 2 to 2. Award *C* and *E* each ½ point.

There are 2 voters who prefer *D* to *E* and 2 prefer *E* to *D*. Thus, *D* and *E* tie by a score of 2 to 2. Award *E* and *D* each ½ point.

	A	*B*	*C*	*D*	*E*
	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
	0	0	1	1	$\frac{1}{2}$
	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Total	$1\frac{1}{2}$	1	3	$2\frac{1}{2}$	2

15. Election 1

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*A*
Second	*B*	*C*	*C*	*C*	*C*
Third	*C*	*A*	*A*	*B*	*B*

Since Candidates *A* and *B* both have the same (high) number of last-place votes, they are both eliminated, leaving Candidate *C* as the winner using Coombs rule. But the winner becomes *A* if the leftmost voter changes his or her ballot as the following shows.

Election 2

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*B*	*A*	*A*
Second	*C*	*C*	*C*	*C*	*C*
Third	*B*	*A*	*A*	*B*	*B*

B has the most last-place votes, thus Candidate *B* is eliminated, and we have the following.

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*C*	*C*	*A*	*A*
Second	*C*	*A*	*A*	*C*	*C*

C now has the most last-place votes, thus Candidate *C* is eliminated, and *A* becomes the winner by the Coombs method.

16. Election 1

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*B*	*C*	*C*	*D*
Second	*B*	*A*	*B*	*B*	*B*
Third	*C*	*C*	*A*	*A*	*C*
Fourth	*D*	*D*	*D*	*D*	*A*

A, *B*, and *D* have the fewest first-place votes and are thus eliminated leaving *C* as the winner using the Hare system. But the winner becomes *B* if the leftmost voter changes his or her ballot as the following shows.

Election 2

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*B*	*B*	*C*	*C*	*D*
Second	*A*	*A*	*B*	*B*	*B*
Third	*C*	*C*	*A*	*A*	*C*
Fourth	*D*	*D*	*D*	*D*	*A*

A has the fewest first-place votes and is eliminated.

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*B*	*B*	*C*	*C*	*D*
Second	*C*	*C*	*B*	*B*	*B*
Third	*D*	*D*	*D*	*D*	*C*

D now has the fewest first-place votes and is eliminated

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*B*	*B*	*C*	*C*	*B*
Second	*C*	*C*	*B*	*B*	*C*

C now has the fewest first-place votes and is eliminated, leaving *B* as the winner.

17. Election 1

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*A*	*C*	*C*	*B*
Second	*B*	*B*	*A*	*A*	*C*
Third	*C*	*C*	*B*	*B*	*A*

Since *A* and *C* have the most number of first-place votes, *B* is eliminated.

Rank	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*A*	*A*	*C*	*C*	*C*
Second	*C*	*C*	*A*	*A*	*A*

Since *C* has the most number of first-place votes, the winner using the plurality runoff rule is *C*. But the winner becomes *B* if the leftmost voter changes his or her ballot as the following shows.

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17. continued

Election 2

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*B*	*A*	*C*	*C*	*B*
Second	*A*	*B*	*A*	*A*	*C*
Third	*C*	*C*	*B*	*B*	*A*

Since *B* and *C* have the most number of first-place votes, *A* is eliminated.

	Number of voters (5)				
Rank	**1**	**1**	**1**	**1**	**1**
First	*B*	*B*	*C*	*C*	*B*
Second	*C*	*C*	*B*	*B*	*C*

Since *B* has the most number of first-place votes, the winner using the plurality runoff rule is *B*.

18. Election 1

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*A*	*B*	*C*
Second	*B*	*C*	*A*
Third	*C*	*A*	*B*

In sequential pairwise voting with the agenda *A*, *B*, *C*, we first pit *A* against *B*. There are 2 voters who prefer *A* to *B* and 1 prefers *B* to *A*. Thus, *A* wins by a score of 2 to 1. *B* is therefore eliminated, and *A* moves on to confront *C*.

There is 1 voter who prefers *A* to *C* and 2 prefer *C* to *A*. Thus, *C* wins by a score of 2 to 1.

Thus, *C* is the winner by sequential pairwise voting with the agenda *A*, *B*, *C*. But the winner becomes *B* if the leftmost voter changes his or her ballot as the following shows.

Election 2

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*B*	*B*	*C*
Second	*A*	*C*	*A*
Third	*C*	*A*	*B*

In sequential pairwise voting with the agenda *A*, *B*, *C*, we first pit *A* against *B*. There is 1 voter who prefers *A* to *B* and 2 prefer *B* to *A*. Thus, *B* wins by a score of 2 to 1. *A* is therefore eliminated, and *B* moves on to confront *C*.

There are 2 voters who prefer *B* to *C* and 1 prefers *C* to *B*. Thus, *B* wins by a score of 2 to 1.

Thus, *B* is the winner by sequential pairwise voting with the agenda *A*, *B*, *C*.

19.

	Number of voters (3)		
Rank	**1**	**1**	**1**
First	*A*	*C*	*B*
Second	*B*	*A*	*D*
Third	*D*	*B*	*C*
Fourth	*C*	*D*	*A*

(a) For *B* to win, consider the agenda *D*, *A*, *C*, *B*.

In sequential pairwise voting with the agenda *D*, *A*, *C*, *B*, we first pit *D* against *A*. There is 1 voter that prefers *D* to *A* and 2 prefer *A* to *D*. Thus, *A* wins by a score of 2 to 1. *D* is therefore eliminated, and *A* moves on to confront *C*.

There is 1 voter who prefers *A* to *C* and 2 prefer *C* to *A*. Thus, *C* wins by a score of 2 to 1. *A* is therefore eliminated, and *C* moves on to confront *B*.

There is 1 voter who prefers *C* to *B* and 2 prefer *B* to *C*. Thus, *B* wins by a score of 2 to 1.

Thus, *B* is the winner by sequential pairwise voting with the agenda *D*, *A*, *C*, *B*.

(b) For *C* to win, consider the agenda *B*, *D*, *A*, *C*.

In sequential pairwise voting with the agenda *B*, *D*, *A*, *C*, we first pit *B* against *D*. There are 3 voters that prefer *B* to *D* and 0 prefer *D* to *B*. Thus, *B* wins by a score of 3 to 0. *D* is therefore eliminated, and *B* moves on to confront *A*.

There is 1 voter who prefers *B* to *A* and 2 prefer *A* to *B*. Thus, *A* wins by a score of 2 to 1. *B* is therefore eliminated, and *A* moves on to confront *C*.

There is 1 voter who prefers *A* to *C* and 2 prefer *C* to *A*. Thus, *C* wins by a score of 2 to 1.

Thus, *C* is the winner by sequential pairwise voting with the agenda *B*, *D*, *A*, *C*.

(c) For *D* to win, consider the agenda *B*, *A*, *C*, *D*.

In sequential pairwise voting with the agenda *B*, *A*, *C*, *D*, we first pit *B* against *A*. There is 1 voter that prefers *B* to *A* and 2 prefer *A* to *B*. Thus, *A* wins by a score of 2 to 1. *B* is therefore eliminated, and *A* moves on to confront *C*.

There is 1 voter who prefers *A* to *C* and 2 prefer *C* to *A*. Thus, *C* wins by a score of 2 to 1. *A* is therefore eliminated, and *C* moves on to confront *D*.

There is 1 voter who prefers *C* to *D* and 2 prefer *D* to *C*. Thus, *D* wins by a score of 2 to 1.

Thus, *D* is the winner by sequential pairwise voting with the agenda *B*, *A*, *C*, *D*.

Note: In any of the three parts, the first two candidates can be switched and the outcome will be the same.

20. If the system fails to satisfy the Pareto condition, then we can choose a sequence of ballots in which every voter prefers *A* to *B*, but *B* is the winner. But now if every voter changes his or her ballot by moving *A* to the top, then this group has achieved an election outcome - namely *A*, by unanimity -- that everyone in the group prefers.

21. Election 1

22%	**23%**	**15%**	**29%**	**7%**	**4%**
D	*D*	*H*	*H*	*J*	*J*
H	*J*	*D*	*J*	*H*	*D*
J	*H*	*J*	*D*	*D*	*H*

D has 22% + 23% = 45% of the first-place votes. *H* has 15% + 29% = 44% of the first-place votes. *J* has 7% + 4% = 11% of the first-place votes. Since *D* has the most first-place votes, Alfonse D'Amato (*D*) is the winner by plurality voting. The plurality rule is group manipulable as the following shows if the voters in the 7% group all change their ballots.

Election 2

22%	**23%**	**15%**	**29%**	**7%**	**4%**
D	*D*	*H*	*H*	*H*	*J*
H	*J*	*D*	*J*	*J*	*D*
J	*H*	*J*	*D*	*D*	*H*

D has 22% + 23% = 45% of the first-place votes. *H* has 15% + 29% + 7%= 51% of the first-place votes. *J* has 4% of the first-place votes. Since *H* has the most first-place votes, Elizabeth Holtzman (*H*) is the winner by plurality voting.

22. (a) Assume that the winner with the voting paradox ballots is *A*. Consider the following two elections:

Election 1

Rank	**Number of voters (3)**		
First	*A*	*B*	*C*
Second	*B*	*C*	*A*
Third	*C*	*A*	*B*

Election 2

Rank	**Number of voters (3)**		
First	*A*	*C*	*C*
Second	*B*	*B*	*A*
Third	*C*	*A*	*B*

In Election 1, the winner is *A* (our assumption in this case) and in Election 2, the winner is *C* (because we are assuming that our voting system agrees with Condorcet's method when there is a Condorcet winner, as *C* is here). Notice that the voter in the middle, by a unilateral change in ballot, has improved the election outcome from his or her third choice to being his or her second choice. This is what that voter set out to do and is the desired instance of manipulation.

(b) Assume that the winner with the voting paradox ballots is *B*. Consider the following two elections:

Election 1

Rank	**Number of voters (3)**		
First	*A*	*B*	*C*
Second	*B*	*C*	*A*
Third	*C*	*A*	*B*

Election 2

Rank	**Number of voters (3)**		
First	*A*	*B*	*A*
Second	*B*	*C*	*C*
Third	*C*	*A*	*B*

In Election 1, the winner is *B* (our assumption in this case) and in Election 2, the winner is *A* (because we are assuming that our voting system agrees with Condorcet's method when there is a Condorcet winner, as *A* is here). Notice that the voter on the right, by a unilateral change in ballot, has improved the election outcome from his or her third choice to being his or her second choice. This is what that voter set out to do and is the desired instance of manipulation.

23. Properties 1, 2, and 3.

24. Properties 1 and 4.

25. Properties 1, 2, and 4.

26. Consider the following scenario: The chair votes for *A* and I vote for *C*. If you vote for *B*, the winner is *A* (your least preferred outcome) while the winner is *C* if you vote for *C*. This shows that voting for *B* does not weakly dominate your strategy of voting for *C*.

27. Consider the following scenario: The chair votes for *A* and I vote for *B*. If you vote for *C*, the winner is *A* (your least preferred outcome) while the winner is *B* if you vote for *B*. This shows that voting for *C* does not weakly dominate your strategy of voting for *B*.

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