Talk abstracts

Speaker: Alexander J. Barrios (Purdue University)
Title: Semistable Conditions on Elliptic Curves
Abstract: The Frey curve $y^2 = x(x+a)(x-b)$ for relatively prime integers a and b comes equipped with an easily computable global minimal model. This global minimal model was essential in the proof of Fermat's Last Theorem as well as used to study Szpiro's conjecture. In this talk, we will review the history of the Frey curve and define the modified Szpiro conjecture, which is equivalent to the ABC conjecture. Afterward, we will mention a classical result due to Frey, Flexor, and Oesterlé pertaining to the primes at which an elliptic curve can have additive reduction. We will outline a new effective proof of this result which allows us to extend the ideas of the Frey curve to all rational elliptic curves with non-trivial torsion subgroup. Time permitting, we will discuss corollaries of this result to the explicit modified Szpiro conjecture and Tate's algorithm.

Speaker: Jacob Bond (Purdue University)
Title: The Monodromy Group of a Composition of Belyĭ Maps
Abstract: Given a Belyĭ map γ and a dynamical Belyĭ map β, $\beta \circ \gamma$ is again a Belyĭ map and $\operatorname{Mon} \beta \circ \gamma \leq \operatorname{Mon} \gamma \wr \operatorname{Mon} \beta$. From the permutation triples corresponding to β and γ, along with some information about paths between edges of β, an explicit description of $\operatorname{Mon} \beta \circ \gamma$ will be presented. Moreover, generators of $\ker(\operatorname{Mon} \beta \circ \gamma \to \operatorname{Mon} \beta)$ can be efficiently computed, allowing for identification of $\operatorname{Mon} \beta \circ \gamma$ from the recognition theorem for semidirect products. Replacing $\operatorname{Mon} \gamma$ in the wreath product by F_2 allows for the determination of $\operatorname{Mon} \beta \circ \gamma$, for any γ, by specialization.

Speaker: Abbey Bourdon (Wake Forest University)
Title: Sporadic Points on Modular Curves
Abstract: Our work is motivated by the following classification problem: For a fixed positive integer d, what finite groups arise as the torsion subgroup of an elliptic curve defined over a number field of degree d? In 1977, Mazur answered this question for elliptic curves over the rational numbers, and the classification for elliptic curves over quadratic fields was completed in 1992 through a series of papers by Kamienny, Kenku, and Momose. A few years later, Merel proved his celebrated Uniform Boundedness Theorem, which implies that if we fix d, then there are only finitely many groups that arise as the torsion subgroup of an elliptic curve defined over number field of degree d. However, the complete list of the groups that arise is unknown for any $d > 2$.

A serious challenge in attempting to extend the classification is the need to identify all groups which arise for only finitely many isomorphism classes of elliptic curves–a phenomenon that does not occur for $d = 1$ or $d = 2$. The known examples correspond to elliptic curves with a rational point of order N appearing in unusually low degree; that is, they correspond to sporadic points on the modular curve $X_1(N)$. In this talk, I will discuss recent results concerning sporadic points of $X_1(N)$ which arise from elliptic curves with j-invariant of bounded degree. This is joint work with zlem Ejder, Yuan Liu, Frances Odumodu, and Bianca Viray.

Speaker: Huy Dang (University of Virginia)
Title: Connectedness of the moduli space of Artin-Schreier curves
Abstract: In this talk, the connectedness of the moduli space of Artin-Schreier curves with fixed genus over an algebraically closed field will be discussed. Pries and Zhu introduce a combinatorial description that partitions the moduli space into irreducible strata and tells us partially how they fit together within the moduli space. We continue their work of studying the relations between the geometry of the strata and their combinatorial data. As an application, when the characteristic is equal to 3, the moduli space is connected for every possible genus. When the characteristic is greater than 3, we show that the moduli space is connected when the genus is sufficiently large, and the bound depends on the characteristic.

Speaker: Sarah Frei (University of Oregon)
Title: Moduli spaces of sheaves on a K3 surface and Galois representations
Abstract: For a fixed K3 surface, we can study various moduli spaces of sheaves having a given topological type. I will discuss a new result about the geometry and arithmetic of these moduli spaces when the K3 surface is defined over an arbitrary field. For any two such moduli spaces of the same dimension, their cohomology groups are isomorphic Galois representations. In particular, when the K3 surface is defined over a finite field, this implies that the moduli spaces have the same zeta functions.

Speaker: Jonathan Gerhard (University of Michigan)
Title: A fun connection between class groups, conjugacy classes, and isogeny classes
Abstract: Let f be the characteristic polynomial of Frobenius of an abelian variety of odd prime dimension p over a finite field; we use f to relate three seemingly disjoint objects. First, we consider the factorizations of primes

in Split(f), a degree $2p$ number field K. Second, we use a parametrization of Shinoda (1980) to describe certain conjugacy casses of the matrix group $\mathrm{GSp}_{2g}(\mathbb{F}_q)$. Our main result (following Gekeler (2003) and Achter and Williams (2015)) is a product formula relating the class number of K to the relative densities of conjugacy classes of $\mathrm{GSp}_{2g}(\mathbb{F}_q)$. Finally, we give a (conjectural) application of our formula to the size of isogeny classes of certain abelian varieties of odd prime dimension.

Speaker: Edray Goins (Purdue University)
Title: Toroidal Belyĭ Pairs, Toroidal Graphs, and their Monodromy Groups
Abstract: A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0,\, 1,\, \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection: $\beta^{-1}\big([0,1]\big) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$. Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. Since $E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$ is a torus, we call (E, β) a toroidal Belyĭ pair. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}\big([0,1]\big) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$.

This project seeks to create a database of such Belyĭ pairs, their corresponding Dessins d'Enfant, and their monodromy groups. For each positive integer N, there are only finitely many toroidal Belyĭ pairs (E, β) with $\deg \beta = N$. Using the Hurwitz Genus formula, we can begin this database by considering all possible degree sequences $\mathcal{D}$ on the ramification indices as multisets on three partitions of N. For each degree sequence, we compute all possible monodromy groups $G = \mathrm{im}\,\big[\pi_1\big(\mathbb{P}^1(\mathbb{C}) - \{0,\, 1,\, \infty\}\big) \to S_N\big]$; they are the "Galois closure" of the group of automorphisms of the graph. Finally, for each possible monodromy group, we compute explicit formulas for Belyĭ maps $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ associated to some elliptic curve $E:\ y^2 = x^3 + A\,x + B$. We will discuss some of the challenges of determining the structure of these groups, and present visualizations of group actions on the torus.

This work is part of PRiME (Purdue Research in Mathematics Experience) with Chineze Christopher, Robert Dicks, Gina Ferolito, Joseph Sauder, and Danika Van Niel with assistance by Edray Goins and Abhishek Parab.

Speaker: Wei Ho (University of Michigan)
Title: Noncommutative Galois closures and moduli problems

Abstract: In this talk, we will discuss the notion of a Galois closure for a possibly noncommutative algebra. We will explain how this problem is related to certain moduli problems involving genus one curves and torsors for Jacobians of higher genus curves. This is joint work with Matt Satriano.

Speaker: Seoyoung Kim (Brown University)
Title: The Sato-Tate conjecture and Nagao's conjecture
Abstract: Nagao's conjecture relates the rank of an elliptic surface to a limit formula arising from a weighted average of fibral Frobenius traces, and it is further generalized for smooth irreducible projective surfaces by M. Hindry and A. Pacheco. We show that the Sato-Tate conjecture based on the random matrix model implies Nagao's conjecture for certain twist families of elliptic curves and hyperelliptic curves.

Speaker: Aaron Landesman (Stanford University)
Title: The average size of 2-Selmer groups over function fields
Abstract: In a recent article, Ho, Le Hung, and Ngo showed the average size of 2-Selmer groups of elliptic curves over function fields is 3. We explain a different proof of this fact using the classical Recillas correspondence to relate counting 2-Selmer elements to counting tetragonal curves on Hirzebruch surfaces.

Speaker: Rachel Pries (Colorado State University)
Title: Computations in cohomology: slopes and stars
Abstract: In the first half of the talk, we use the Newton polygon stratification of PEL-type Shimura varieties to compute slopes of Frobenius on crystalline cohomology for curves that are cyclic covers of the projective line. As an application, we give new examples of supersingular curves of genus 5-11. This is joint work with Li, Mantovan, and Tang. In the second half of the talk, we use commutators associated with a star shape to determine a cup product in cohomology. As an application, we determine the action of the absolute Galois group on the second quotient of the lower central series of the fundamental group of the Fermat curve. This is joint work with Davis and Wickelgren.

Speaker: Dane Skabelund (UIUC)
Title: The de Rham cohomology of the smallest Ree curve
Abstract: The Deligne-Lusztig curves of Ree type are a family of curves in characteristic 3 with exceptional arithmetic and geometric properties. In this talk I will describe the structure of the p-torsion of the Jacobian of the smallest

curve in this family, which has genus 3627. This is done by computing the action of the Frobenius and Verschiebung operators on the de Rham cohomology of the curve. This work is joint with Iwan Duurma.

Speaker: Vaidehee Thatte (Queen's University)
Title: Ramification Theory for Arbitrary Valuation Rings in Positive Characteristic
Abstract: In classical ramification theory, we consider extensions of complete discrete valuation rings with perfect residue fields. We would like to study arbitrary valuation rings with possibly imperfect residue fields and possibly non-discrete valuations of rank ≥ 1, since many interesting complications arise for such rings. In particular, defect may occur (i.e. we can have a non-trivial extension, such that there is no extension of the residue field or the value group).
We present some new results for Artin-Schreier extensions of arbitrary valuation fields in positive characteristic p. These results relate the "higher ramification ideal" of the extension with the ideal generated by the inverses of Artin-Schreier generators via the norm map. We also introduce a generalization and further refinement of Kato's refined Swan conductor in this case. Similar results are true in mixed characteristic $(0, p)$.

Speaker: John Voight (Dartmouth College)
Title: Rigorous computation of the endomorphism ring of a Jacobian
Abstract: We describe algorithms for the rigorous computation of the endomorphism ring of the Jacobian of a curve defined over a number field. This is joint work with Edgar Costa, Nicolas Mascot, and Jeroen Sijsling.

Speaker: Mckenzie West (Kalamazoo College)
Title: Solving S-Unit Equations
Abstract: Solutions to S-unit equations have a variety of applications in number theory and arithmetic geometry. Inspired by work of Tzanakis–de Weger, Baker–Wüstholz and Smart, we use the LLL methods available in Sage to implement an algorithm that returns all S units $\tau_0, \tau_1 \in \mathcal{O}_S^\times$ such that $\tau_0 + \tau_1 = 1$.