Math 221
Worksheet: Inverse functions
September 3-4, 2014

1. Find the domain and range of each function, and draw a rough graph. Then find the function's inverse, find its domain and range, and draw this graph on the same set of axes.

   (a) $f(x) = x^3 + 5$

   (b) $g(x) = \sqrt{-1 - x}$

   (c) $h(x) = \sqrt[3]{1 - x^3}$ (What is unusual about this function and its inverse?)

2. Draw a nice graph of $y = \sin(x)$. Notice that, if we restrict our domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the sine function is one-to-one (explain why). Use this to draw a graph of the inverse of $y = \sin(x)$, when the domain of $\sin(x)$ is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$. This inverse is called arcsin, or $\sin^{-1}$. What are its domain and range?

3. Repeat the previous problem with $y = \tan(x)$ instead of $y = \sin(x)$. This time, you choose the domain restriction by looking at the graph of the tangent function.

4. What is...

   (a) $\arcsin(\sin(\frac{\pi}{4}))$

   (b) $\arcsin(\sin(\frac{5\pi}{4}))$

   (c) $\sin(\arcsin(\frac{3}{4}))$

   (d) $\sin(\arctan(\frac{4}{3}))$

5. (a) Let $f(x) = \arcsin(\sin(x))$. What are the domain and range of f? Can you simplify the formula for this function?

   (b) Repeat part (a) with $f(x) = \sin(\arcsin(x))$.