Math 475, Spring Semester 2001-02   NAME:
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Final Exam: (150 points, 15 per problem): Thurs., 16 May, 2002. Total Points:
Be sure to show your work. Binomial coefficients, factorials, ... need not be evaluated.

1. [15 points] On a rectangular grid, there is a pot of gold 8 blocks east and 7 blocks north of where you are standing. You need to walk 15 blocks to claim the gold. The two horizontal and vertical blocks that begin 4 blocks east and 4 blocks north of your current location is under repair and are **not usable**. How many different ways are there for you to walk and claim the gold?
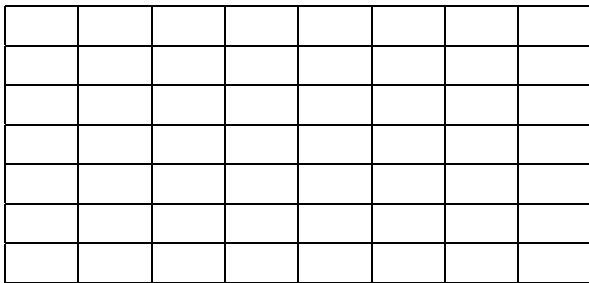
2. [15 points] Consider the permutations of the letters in the "word" BAADD. We want to count permutations of the five letters in this word so that no letter occurs in a position in which it occurs in BAADD.

(a) Formulate this counting problem as a problem of counting non-attacking rooks on a chessboard with forbidden positions.

(b) Solve the forbidden position problem.

3. [15 points]

(a) What is the generating function for the sequence of numbers defined by the recurrence relation

$$h_n - 2h_{n-1} - 3h_{n-2} = 0, (n \geq 2), h_0 = 1, h_1 = 2.$$

(b) Write the generating function for the number of ways to make n dollars if you can only use 1 dollar, 5 dollar, 20 dollar, and 50 dollar bills.

4. [15 points]

(a) What do the Stirling numbers $S(p, k)$ of the second-kind count? Use this to evaluate $S(p, 2)$, explaining your reasoning.

(b) What do the Stirling numbers $s(p, k)$ of the first-kind count? Use this to evaluate $s(p, 2)$, explaining your reasoning.

5. [15 points] (a) List four **different** properties of graphs with n vertices that are trees.

(b) **Grow** a spanning tree for the vertex-edge graph of the 3-cube labelling the edges of the tree in the order $1, 2, 3, \ldots$ in which they are chosen.

6. [15 points] Determine the **chromatic polynomial** and then the **chromatic number** of the graph drawn below.

7. [15 points] State Pascal's formula for binomial coefficients and give a **combinatorial** derivation (interpretation) of it.

8. [15 points] Let $h(n) = n^3 - n^2 + n + 1$. Compute enough of the difference table for $h(n)$ in order to write $h(n)$ as a linear combination of falling factorials $[n]_k$

9. [15 points] A two-sided colored tromino is a 1-by-3 board of three squares where each square on both sides is colored with one of the colors R,B,G,Y, or P How many **different** two-sided colored trominoes are there?

10. [15 points] Compute the number of non-equivalent ways to color the corners of a regular 8-gon with colors R,W, and B under the action of the corner-symmetry group of the 8-gon.