Math 475, Final Exam (125 points) Name:
12:25 pm, May 19, 2000, B115 Van Vleck Hall
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1. (20 points) We have a multiset X of 14 balls, consisting of 3 identical Red, 4 identical Blue, 5 identical Green, and 2 identical Yellow balls.

(a) How many (linear) permutations of X are there?

(b) How many (linear) permutations of X are there if the yellow balls are not consecutive?

(c) How many circular permutations of X are there if the Yellow Balls are opposite one another?

(d) How many ways are there to arrange the balls of X in a 2 by 7 formation if the Yellow Balls are to be in different rows?

2. (10+5=15 points)

(a) Evaluate the following sum: $\sum_{k=1}^{n}(-1)^{k-1}k\binom{n}{k}$ for $n \geq 1$.

(b) Explain the combinatorial reasoning behind the recurrence relation for the Stirling numbers of the second kind:

$$S(p,k) = kS(p-1,k) + S(p-1,k-1) \quad (1 \leq k \leq p-1).$$

3. (15 points) How many ways are there to place 7 non-attacking rooks on the 7 by 7 board with forbidden positions as shown:

X	X					
X	X					
		X	X	X		
		X	X	X		
					X	X

4. (15 points) Identify each of the relations on a set X below as a partial order, equivalence relation, total order, or none of the above:

(a) X the collection of all subsets of $\{1, 2, \ldots, 10\}$ with $A \, R \, B$ iff $A \subseteq B$.

(b) X the set of all real numbers with $A \, R \, B$ iff $|a| = |b|$.

(c) X the set of positive integers with $a \, R \, b$ iff a is a factor of b.

(d) X the set of vertices of a tree T with root r with $a \, R \, b$ iff the chain from r to b in T passes through a.

(e) X the set of ordered pairs (a, b) of real numbers with $(a, b) \, R \, (c, d)$ iff $a \leq c$ (as real numbers) and $b \leq d$.

5. (10 points) Give the ordinary generating function in closed form for the number h_n of solutions in nonnegative integers of the equation $3e_1 + e_2 + 2e_3 + 5e_4 = n$ ($n \geq 0$).

6. (15 points) Determine the chromatic polynomial of the graph:

7. (10 points) Consider the following network N with source s and target t where the numbers on arcs represent their capacities and the numbers in brackets $[\cdot]$ on arcs represent the value of a function f on the arcs of N.

(a) Check that f is a flow from s to t, and determine its value:

(b) Starting with f, use **the basic flow algorithm** and obtain a "flow-augmenting path" from s to t to give a flow f' whose value is one more than the value of f. Is f' a maximum flow? If so, give a cut whose capacity equals the value of f'.

8. (15 points) Use **Burnside's Theorem** to determine the number of inequivalent colorings of the corners of a regular hexagon (6-gon) P in the presence of the full corner symmetry group of P.

9. (10 points) A *marked 4-omino* is a 1 by 4 piece of 4 unit squares joined side to side where each square is marked with 1, 2, 3, 4, 5, or 6 dots. Use **Burnside's Theorem** to determine the number of different marked 4-ominoes.

EXTRA CREDIT PROBLEM (20 points) Consider the graph G with vertices and edges as shown:

(a) How many chains of length 12 connect the lower left vertex X with the upper right vertex Y?

(b) How many such chains are there if the "middle vertex" A and all the edges that touch it are eliminated from the graph?

(c) How many such chains which do not use any of the vertices on the diagonal running from X to Y?