Math 240, Spring Semester 2000-01 NAME:
(Prof.) R.A. Brualdi. (TA) M. Ward
Exam 2: April 25, 2001, 2001 Total Points:

1. **Functions**

(i) Let $A = \{a, b, c, d, e\}$. Draw the digraph of the function $f : A :\rightarrow A$ which has the constant value c on A.

(ii) Let $A = \{a, b, c, d, e\}$. Draw the digraph of the identity function on A.

(iii) Let functions f, g, h, k with domain Z^+ be defined by:

$$f(n) = n^3 - 4n^2 + 3,\ g(n) = n^3 + 5n^2,\ h(n) = \log_2(n),\ \text{and } k(n) = \left\lfloor \frac{n+1}{5} \right\rfloor .$$

Draw the digraph of the relation big-O on $\{f, g, h, k\}$.

(iv) Consider the permutations

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 1 & 2 & 3 & 4 \end{pmatrix} \text{ and } q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 5 & 2 & 1 \end{pmatrix}.$$

(iv-a) Compute $p \circ q$

(iv-b) Compute q^{-1}

(iv-c) Write p as a product of disjoint cycles, **and** as a product of transpositions.

cycles:

transpositions

2. **Partial Orders**

(i) Circle the properties that every finite poset has:

not reflexive – reflexive – symmetric – not symmetric –
antisymmetric – transitive – not transitive –
every pair of elements comparable –
LUB of every pair of elements exists –
maximal elements always exist – a greatest element always exists.

(ii) Consider a six letter alphabet with letters given in order from first to last by u, v, w, x, y, z. Put the following "words" in lexicographic order:

wuxy, wuxx, uxzyz, yxx, zu.

(iii) Draw (carefully!) the Hasse diagram of the poset of all subsets of $A = \{a, b, c\}$ partially ordered by set-inclusion. Be sure to label all the points in the diagram.

(iv) List **ALL** the topological sortings of the poset whose Hasse diagram is:

(v) In the poset of integers $\{1, 2, 3, \ldots, 15\}$ ordered by divisiblity (is a factor of), compute the following or say that it does not exist:

- GLB $\{10, 14\}$:
- GLB $\{7, 9\}$:
- LUB $\{3, 7\}$:
- LUB $\{3, 4\}$:

(vi) Consider the poset of partitions of $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ (i.e. equivalence relations R on A partially ordered by set-inclusion), and the partition $\pi : \{1, 2, 3\}, \{4, 5, 6\}, \{7, 8\}$ with 3 parts. Determine:

(vi-a) a partition less than π with 5 blocks.

(vi-b) a partition greater than π with 2 blocks.

3. **Boolean algebras and Boolean functions**

(i) Is the lattice D_{70} of all positive divisors of 70 a *Boolean algebra*? Explain/Justify your answer.

(ii) Consider the Boolean function f of 3 Boolean variables x, y, z with f having the value 1 only for $x = 1, y = 0, z = 1$, $x = 0, y = 0, z = 1$, and $x = 1, y = 1, z = 1$. Give the function f as a Boolean expression (polynomial).

4. **Trees and Graphs**

(i) Construct the rooted tree corresponding to the algebraic expression:

$$(((a \times b) - c) \times d) \times (e + (f \times g)).$$

(ii) List the elements of the rooted tree in (i) in **preorder**:

(iii) Evaluate the following algebraic expression given in Polish (prefix) form:

$$+ \times \times + (3)(4)(5)(6) + \times (7)(8)(9)$$

(iv) Represent the following ordered tree as a binary positional tree:

(v) Consider the weighted graph below to which we have begun to apply either Prim's algorithm or Kruskal's algorithm (those edges bolded are the ones picked thus far).

(v-a) Which algorithm was applied in this case (or could it have been either)?

(v-b) What is the *next* edge that can be chosen using the algorithm?

(vi) what is the chromatic number and chromatic polynomial of the (undirected) graph which is a cycle with 4 vertices and 4 edges?