Math 240, Spring Semester 2000-01 NAME:
R.A. Brualdi
Exam 1: March 7, 2001 Total Points:

1. Consider the two integers 177 and 141.

(a) Determine their GCD (show your work):

(b) Write the GCD as a linear combination of 177 and 141 (GCD $= a \cdot 177 + b \cdot 141$ with a and b integers)(show your work):

(c) The LCM of 177 and 141 (show your work)?

2. Determine the truth value (True, False, or Need More Information) of the following statements:

(a) If -2 is a positive number, then $\sqrt{-1}$ is a real number.

TRUE **FALSE** **CAN'T SAY**

(b) $\exists y \forall x \; 2x - 3y = 5$ (x and y designate real numbers)

TRUE **FALSE** **CAN'T SAY**

(f) $\neg(\exists x P(x)) \Rightarrow \wedge x(\neg P(x))$

TRUE **FALSE** **CAN'T SAY**

3. Prove using mathematical induction:

$$3|(4^n + 2) \text{ for all integers } n \geq 1.$$

4. N balls are distributed into 3 boxes. The smallest value of N that guarantees that either the first box contains at least 4 balls or the second box contains at least 5 balls or the third box contains at least 6 balls is:

5. Consider the English alphabet of 26 letters including the 5 vowels a, e, i, o, u.

(a) The number of **sets** of 7 letters with exactly 3 vowels is:

(b) The number of different **sequences** of 7 letters containing 7 distinct letters including exactly 3 vowels is:

6. Solve the recurrence relation

$$a_n = 2a_{n-1} + 3a_{n-2} \quad (n \geq 3), a_1 = 1, a_2 = 4.$$

7. An urn contains 5 balls of each of the colors R, W, and B (so 15 altogether). The numbers $-4, -1, 1, 2, 3$ are written on each the balls of each color (each number occurs once with each color). Suppose you reach into the urn and grab 3 balls all at once.

(a) The probability that all the ball have different colors is:

(b) The probability that all the balls have the same color is:

(c) Suppose one makes a game out of the above urn whereby drawing a ball pays you in dollars the number on the ball (negative numbers means you pay). If you play this game, say, a 100 times, you would expect to win how much per game?

8. The matrix of the transitive closure of the relation R on $\{1, 2, 3, 4, 5\}$ in which $1R2$, $1R3$, $1R5$, $2R3$, $3R4$, $4R2$, $5R1$ is

9. For each of the following two relations, determine whether they are reflexive, irreflexive, symmetric, antisymmetric, and transitive. If an equivalence relation, determine the partition into equivalence classes.

(a) R the relation whose matrix is

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Reflexive ... Irreflexive ... Symmetric ... Antisymmetric ... Transitive

Equivalence classes (if applicable) are:

(b) the relation R on the set of integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ where aRb if and only if $a = b \pm 1$.

Reflexive ... Irreflexive ... Symmetric ... Antisymmetric ... Transitive

Equivalence classes (if applicable) are: