

Fall Semester, 2002-03

**Math 743: Exercises 4; Due Friday, December 13, 2002.**

1. Let  $A$  be a *reducible* doubly stochastic matrix of order  $n \geq 2$ . Prove that the rows and columns of  $A$  can be permuted to obtain a nontrivial direct sum of doubly stochastic matrices.

2. Let  $A \geq O$  be a matrix of order  $n$ . Then  $A$  is called *doubly substochastic* provided all row and column sums are at most 1. Prove that there exists a doubly stochastic matrix  $B$  such that  $A \leq B$  (entrywise).

3. Let  $A \geq O$  be a matrix of order  $n$ . Then  $A$  is called *doubly superstochastic* provided all row and column sums are at least 1. Find an example of a doubly superstochastic matrix  $A$  of order 3 for which there does not exist a doubly stochastic matrix  $B$  with  $B \leq A$ .

4. Show that the elementary doubly stochastic matrices of order 3 do not generate the multiplicative semigroup of doubly stochastic matrices of order 3 by showing that

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

cannot be written as a product of elementary doubly stochastic matrices.