Fall Semester, 2002-03

Math 743: Exercises 4; Due Wednesday, November 27, 2002.

1. Let $A \geq O$ be an irreducible matrix of order n with a positive main diagonal, and let $0 \neq x \geq 0$ be a vector of size n. Prove that if x has at least one 0 then Ax has fewer 0's than x.

2. Is the product of two irreducible nonnegative matrices of order n irreducible? Proof or Counterexample.

3. Let $A \geq O$ be a matrix of order n, and let B be a principal submatrix of A. Prove that $\rho(B) \leq \rho(A)$. Does this hold for arbitrary matrices? Counterexample??

4. Let $A \geq O$ be a matrix of order n. Suppose that the sum of the entries in each row is a constant c. Prove that $\rho(A) = c$. Use this to show that in general, if A has minimal row sum m and maximal row sum M, then $m \leq \rho(A) \leq M$. (Similar for column sums.)