Fall Semester, 2002-03

Math 743: Exercises 3; Due Wednesday, November 13, 2002.

1. Compute explicitly the additive compound $A^{[2]} = A^{(2,1)}$ of a square matrix of order 4.

2. If the m by n matrix has rank r prove that the rank of its rth compound $A^{(r)}$ is 1.

3. Prove that a positive semi-definite matrix of rank r is the sum of r positive semi-definite matrices of rank 1. What is the form of a positive semi-definite matrix of rank 1?

4. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two square matrices of order n. Then their *Hadamard product* is the entry-wise product matrix $A \circ B = [a_{ij}b_{ij}]$ of order n. Prove that if A and B are positive semi-definite (resp. positive definite) hermitian matrices then so is $A \circ B$. Hint: Look for $A \circ B$ in $A \otimes B$.