Fall Semester, 2002-03

Math 743: Exercises 1; Due Wednesday, Sept. 25.

1, Let A be a matrix of order n and let μ be an eigenvalue of A. Prove that the largest size of a Jordan block of A for μ (called the *index* of μ) equals the smallest positive integer k such that

$$\text{rank }(A - \mu I)^k = \text{rank }(A - \mu I)^{k+1}.$$

2. Let

$$A = \begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} K & L & F_1 \\ M & N & F_2 \\ G_1 & G_2 & H \end{bmatrix}$$

where E is a nonsingular matrix of order k and K is a nonsingular submatrix of order $t < k$. Prove that Schur complements satisfy the following quotient rule:

$$A/E = (A/K)/(E/K).$$