Math 641, Fall 1999

R.A. Brualdi

Exercise Set 4, * exercises due Friday, October 29, 1999

Let C be an $[n, k, d]$ code. The *support* of C is the number of coordinates where not all codewords of C equal zero (it is the number of nonzero columns of a generator matrix of C). For $1 \le r \le k$ the rth *generalized Hamming weight* of C is the minimum support of a r-dimensional subcode of C. So d_1 is the minimum support of a 1-dimensional subcode, i.e. the minimum support of a non-zero vector in C, i.e. the minimum weight of a nonzero codeword. The numbers $d_1, d_2, \ldots, d_k$ are called the *weight heirarchy* of C.

* 1. Prove

$$d = d_1 < d_2 < \cdots < d_k \le n.$$

* 2. Prove the generalized Singleton bound

$$d_r \le n - k + r \quad (1 \le r \le k),$$

and show that MDS codes meet this bound for all r.

* 3. Determine the weight heirarchy of the $[15, 4, 8]$ binary simplex code and the Reed-Muller code $RM(1, 4)$