

**Corrections and Comments for the 5th edition of: “Introductory
Combinatorics”
by Richard A. Brualdi
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(Other corrections/comments gratefully received. Mail to: brualdi AT math.wisc.edu)

1. page viii, middle of page: Section 3.6 should be Section 2.6. (Thanks to Balázs Boros.)
2. page x (second page of the Preface), lines -8 and -12 : both references to Section 5.7 should be to Section 5.6. (Thanks to Edmond Lee, Nova Univ.)
3. Page 16 (the mutually overlapping circles problem): The statement: “Each of the $2(n - 1)$ arcs divides a region formed by the first $n - 1$ circles $\gamma_1, \dots, \gamma_{n-1}$ into two, creating $2(n - 1)$ more regions” is not correct. It may happen that a region gets divided into more than two regions. It should say that each of the $2(n - 1)$ arcs creates a new region, giving $2(n - 1)$ more regions. (This problem and solution has been in various places in the book since its first edition. This is the first time that this subtle error has been brought to my attention! Thanks to Cristina Ballantine)
4. Page 30, line -10 : The \mathcal{S} should be A . (Thanks to Reza Kahkeshani)
5. page 32 (end of second paragraph): The statement “We will return to this example in Section 3.5.” should be “We will return to this example in Section 2.5.” (Thanks to Shuyang Fang)
6. page 33, line 13: “next two sections” should be “next four sections”. (Thanks to Balázs Boros.)
7. Page 56, third/fourth line of Example: “eight ordered triples” not “eight ordered pairs”. Thanks to Edmond Lee of Nova University)
8. Page 62, Exercise 16: The reference should be to Theorem 2.3.1 (not Theorem 3.3.1). (Thanks to Cristina Ballantine)

9. Page 63, Exercise 23: This exercise may be a little ambiguous. The intent was to determine the number of ways the players could have their sets of 13 cards as the game begins. (Thanks to Doug Shaw)
10. Page 63, Exercises 24 and 25: There may be some ambiguity here. It would be better to say: “In how many ways can the people be seated?” (Thanks to Doug Shaw)
11. Page 66, Exercise 45: Replace “each of which holds at least twenty books” with “each of which holds at most twenty books” . (Thanks to Henry Cryns)
12. Page 67, Exercise 59: Reference should be to Section 2.6, not Section 3.6. (Thanks to Kevin Burns and later Edmund Lee.)
13. Page 79, Proof of Theorem 3.3.1 (Ramsey’s Theorem). On lines –11 and –10 it says “Consider one of the points x of K_n ” when it should say “Consider one of the points x of K_p ”. (Thanks to Jeff Norden)
14. Page 80, line –4: Relace the colon with a period. (Thanks to Balázs Boros.)
15. Page 81, line-15: It should say $K_{49} \rightarrow K_5, K_5$ and not $K_{59} \rightarrow K_5, K_5$. (Thanks to Shuyang Fang, Dartmouth)
16. Page 83, Exercise 4: Replace “if $n + 1$ integers are chosen” with “if $n + 1$ distinct integers are chosen”. (Thanks to Yucheng Qiu)
17. Page 105, line 9: Replace the colon with a period. (Thanks to Balázs Boros.)
18. Page 112, line 9: The last permutation of $\{1, 2, 3\}$ generated should be 213 (not the 312 in the last position of the first column). (Thanks to Yu Zhang, Fudan Univ.)
19. Page 107, line –9: Replace “Gray of order” with “Gray code of order”. (Thanks to Balázs Boros.)
20. Page 108, line -10: Replace “5is” with “5 is”. (Thanks to Balázs Boros.)
21. Page 109, line –12: Replace “all the 4-subsets” with “all the subsets”. (Thanks to Balázs Boros.)

22. Page 113, line 2: “ r -rubsets” should be “ r -subsets”. (Thanks to Edmond Lee, Nova Univ.)
23. Page 117, line 16: Replace “ i is smaller than j ” with “ i is smaller than or equal to j ”. (Thanks to Balázs Boros.)
24. Page 117, line 17: Change (X, \leq) to $(X, |)$. (Thanks to Reza Kahkeshani)
25. Page 129, line 1: Theorem 3.3.2 should be Theorem 2.3.4. (Thanks to Jeremy Lovejoy)
26. Page 142, line 8: Replace $\{i_1.i_j\}$ with $\{i, j\}$. (Thanks to Balázs Boros.)
27. Page 142, lines -15, -14, -13: That sentence should read: More generally, given any $A \subset S$ with $|A| = k$, the number of maximal chains containing A equals $k!(n - k)! \dots$. (Thanks to Michelle Bodnar, University of Michigan, Class of 2013, and to Kevin Burns)
28. Page 143, end of proof of Theorem 5.3.3: it should have been $|\mathcal{A}| = \sum_{k=0}^n \alpha_k$, although it is not incorrect as stated. (Thanks to Anna Shors, U. Rochester Class of 2020.)
29. Page 144, line 1: section 3.4 should be Section 2.4. (Thanks to Jeremy Lovejoy.)
30. Page 146, line 11: Reference should be to Section 2.5 (not Section 3.5 as written). Thanks to Seth Chi Shing.)
31. Page 149, line 1 of Section 5.6: Replace “In Section 5.4” with “In Section 5.3”. (Thanks to Balázs Boros.)
32. Page 152, middle of page: Replace “Each chain will have to contain exactly one subset of X of size $\binom{n}{\lfloor \frac{n}{2} \rfloor}$ ” with “Each chain will have to contain exactly one subset of X of size $\lfloor \frac{n}{2} \rfloor$ ”. (Thanks to Mike Stemper)
33. Page 156, Exercise 19: “observing” here is meant to imply that a student should “prove.” So maybe it would have been better to say: “by first proving that $m^2 = 2\binom{m}{2} + \binom{m}{1}$.” (Thanks to Doug Shaw)

34. Page 157, Exercise 26: The identity is incorrect. It should read:

$$\sum_{k=1}^n \binom{n}{k} \binom{n}{k-1} = \frac{1}{2} \binom{2n+2}{n+1} - \binom{2n}{n}.$$

(Thanks to an unknown Chinese student)

35. Page 159, Exercise 44: The value of the summation should be $(-3)^n$;

$$\sum_{n_1+n_2+n_3=n} \binom{n}{n_1 \ n_2 \ n_3} (-1)^{n_1-n_2+n_3} = (-3)^n.$$

(Thanks to Moa Apagodu and his student Heather Smith)

36. Page 159, Exercise 45: The summation should be $(-4)^n$ (not 0. (thanks to Huafei Yan).

37. Page 159, Exercises 44 and 45: Other variations of the identities in these exercises are:

$$\sum_{n_1+n_2+n_3=n} \binom{n}{n_1 \ n_2 \ n_3} (-1)^{n_2} = 1$$

$$\sum_{n_1+n_2+n_3+n_4=n} \binom{n}{n_1 \ n_2 \ n_3 \ n_4} (-1)^{n_2+n_4} = 0.$$

(Thanks to Donald Kreher, and independently Rod Peled.)

38. Page 161, line -12: The reference to Chapter 3 should be to Chapter 2. (Thanks to Michael Barrus)

39. Page 169, Last paragraph, first sentence: A closing paranthesis is missing (or (10.a, 10.b, 10.c)). Thanks to Joaquin Padilla Montani.

40. Page 170, Line 1: The reference should be to Theorem 2.5.1, and not to Theorem 3.5.1. (Thanks to Christopher White)

41. Page 178, line -12: Replace “As seen in Section 3.4” with “As seen in Section 2.4”. (Thanks to Balázs Boros.)

42. Page 214, line 6: The k in the formula for g_n should be a p :

$$g_n = \sum_{p=0}^{n-1} \binom{n-1-p}{p}.$$

(Thanks to Stephanie Vance.)

43. Page 216, line –11: Replace “see Section 5.6” with “see Section 5.5”.
(Thanks to Balázs Boros.)
44. Page 216, line –11: Replace “From Chapter 3” with “From Chapter 2”. (Thanks to Balázs Boros.)
45. Page 216, Footnote: Should be Section 5.5 not 5.6. (Thanks to Edmond Lee.)
46. Page 219, Line 5: Replace “bags of fruit” with “bags of n fruits”.
(Thanks to Jeremy Lovejoy)
47. Page 223, line 10: The reference should be to Section 7.2, not 7.5.
(Thanks to Sultan M. Al-Suleiman and independently Cristina Ballantine)
48. Page 223, line –9: Replace “Section 3.4” with “Section 2.4”. (Thanks to Balázs Boros.)
49. Page 223, line -2: The reference should be to Section 2.4, not Section 2.3. (Thanks to Michael D. Barrus)
50. Page 224, line -4: The reference should be to Section 2.4 not Section 3.4. (Thanks to Susan Ydstie.)
51. Page 224, last line: It should be $\{m_1 \cdot a_1, m_2 \cdot a_2, \dots, m_k \cdot a_k\}$, not $\{m_1 \cdot e_1, m_2 \cdot e_2, \dots, m_k \cdot e_k\}$. (Thanks to Cristina Ballantine)
52. Page 226, displayed equations: $g^{(e)}$ should be $g^{(e)}(x)$. (Thanks to Jeremy Lovejoy)
53. Page 226, near the bottom: Replace $h_n = 1 + 3 + 3^2 + \dots + 3^{n-1} \dots$ with $h_n = 2 + 3 + 3^2 + \dots + 3^{n-1} \dots$. (Thanks to Bernard Lidicky.)

54. Page 234, line 11: Replace “Section 5.6” with “Section 5.5”. (Thanks to Balázs Boros.)
55. Page 235, line 10: There is a missing 0 in the displayed equation. It should read $h_n - 5h_{n-1} + 6h_{n-2} = 0$ ($n \geq 2$). (Thanks to Edmond Lee.)
56. Page 236, line -12: The coefficient of x^2 in the expansion of $g(x)$ is -16 (not -15). (Thanks to Michael Barrus)
57. Page 238, line 4 from bottom: In the last displayed equation the second c_2 should be c_3 . (Thanks to Brian Gordon,)
58. Page 245, line -8 : Replace “ $= 2^{n-1}(h_0+1)+$ ” with “ $= 2^{n-1}(2h_0+1)+$ ”. (Thanks to Balázs Boros.)
59. Page 246, line -3 : Replace “ $-2xg(x) =$ ” with “ $2xg(x) =$ ”. (Thanks to Balázs Boros.)
60. Page 250, line -10 : Replace “with $r = 3$ ” with “ $r = 2$ and $r = 3$ ”. (Thanks to Balázs Boros.)
61. Page 253, Line 3 from the top: An x^n is missing. It should be $\sum_{n=0}^{\infty} (n+2)3^n x^n$. (Thanks to Bernard Lidicky.)
62. Page 256, line -2 : Replace “end of Section 5.6” with “end of Section 5.5”. (Thanks to Balázs Boros.)
63. Page 262, Exercise 49: In front of each term of the summation on the right-hand side of the identity, there should be the factor $q^{\binom{k}{2}}$. Thanks to Albert Shih.
64. Page 264, Exercise 51: The reference should be to Section 7.5 (not to Section 7.6). (Thanks to James Sellers.)
65. Page 266. Theorem 8.1.1: Replace “whose partial sums are always positive” with “whose partial sums are always nonnegative”. (Thanks to Jeremy Lovejoy)
66. Page 267, line -11: ”The number of sequences of $(n+1)$ $+1$ s and $(n+1)$ -1 s is the number” should be ”The number of sequences of $(n+1)$ $+1$ s and $(n-1)$ -1 s is the number”. Thanks to Chenglin Chen.

67. Page 273, equation (8.7): Replace $C_1C_{n-2}C_1$ with C_1C_{n-2} . (Thanks to Balázs Boros.)
68. Page 275, line -5: Replace “ $+a_1n+a_0$ ” with “ $a_1(n+1)+a_0$ ”. (Thanks to Jeremy Lovejoy)
69. Page 280, line 1: The reference should be to equation (5.19) not to equation (5.14). (Thanks to Craig Rasmussen.)
70. Page 282, line 2: Replace “see Section 3.2” with “see section 2.2”. (Thanks to Balázs Boros.)
71. Page 285, line 5 from bottom: it should be type (1) not type (i). (Thanks to Alfred Geisel.)
72. Page 286, line 9: it should be partitions of $\{1, 2, \dots, p\}$, not $\{1, 2, \dots, k\}$. (Thanks to Alfred Geisel.)
73. Page 286, line 16-17: The set $\{1, 2, \dots, k\}$ should be $\{1, 2, \dots, p\}$; so it is partitions of $\{1, 2, \dots, p\}$ into k nonempty, distinguishable boxes that are being counted. (Thanks to Tyson Williams,)
74. Page 293, line right before Example: There is a subscript j missing on n in the display. It should read
- $$n_i^* = |\{j : n_j \geq i\}| \quad (i = 1, 2, \dots, l).$$
- (Thanks to Brian Gordon)
75. Page 293, line -9: Replace “upper right corner” with “upper left corner”. (Thanks to Balázs Boros.)
76. Page 319, Exercise 37: The initial C_n should be R_n : “The large Schröder number R_n counts” (Thanks to Stephanie Vance.)
77. Page 325, paragraph beginning with “The discussion in ...”. There is the assertion: “There is a tiling of the board if and only if the domino family has an SDR.” This assertion should have begun with: Assume $m = n$. (Thanks to Donald Kreher)
78. Page 367, line -6: ”nir” should be ”nine”. (Thanks to Ricardo Cervantes.)

79. Page 368. in the sixth column of the display on the top of the page, the last triple $\{6, 7, 10\}$ should be $\{6, 7, 1\}$. (Thanks to Ricardo Cervantes.)
80. Page 391, Exercise 36, part (3): the two occurrences of 12 should be 13, that is, "The 13 blocks developed from B_1 together with the 13 blocks developed from B_2 are the blocks of a Steiner triple system of index 1 with 13 varieties." (Thanks to Amirbehshad Shahrashbi.)
81. Page 409, last line before footnote: "ggraph" should be "graph". (Thanks to Michael Barrus)
82. Page 416, line 5: the text refers to Theorem 11.1.1 but should instead refer to Theorem 11.2.2. (Thanks to Michael Barrus)
83. Pages 417–418: The algorithm given to construct a Hamilton cycle in a graph satisfying the Ore condition needs a slight modification. After applying step (2) (iii) the path is altered. So one should return to step (1) get a longer and longer path until it is not possible to make it any longer. (Thanks to Michelle Bodnar.)
84. Page 422, line 6: The walk α_1 from x to a should be from x to z : $\alpha_1 : x - \cdots - z$. (Thanks to Doug Shaw)
85. Page 430, lines 11–12: There is a 'non' missing. It should read: As we have argued, the terminal graph $T=(V,F)$ is connected and does not have any non-bridges; hence it is a tree. [Of course it would be better to say that each edge is a bridge.] (Thanks to Jackson Criswell of Central Michigan University.)
86. Page 432, footnote # 44: It's Claude Shannon (not Clause). (Thanks to Donald Kreher and his student Eric Crawley)
87. Page 438, line 7: "In Section 11.5, describe a ..." should be "In Sections 11.5, we described ,,, ". (Thanks to Michael Barrus)
88. Page 451, Exercises 16 and 17: There are two part (b)'s in these exercises. The second (b) in each case should have been (c). In the case of the second (b) in Exercise 16, it says: "Determine all the nonisomorphic subgraphs of order 6." Of course, these are the same as in the first (b). It would have been better to ask: "Determine all the nonisomorphic subgraphs of order 4." (Thanks to Doug Shaw)

89. Page 453, Exercise 34: The intent here was that the graphs are connected, since an Eulerian graph may have vertices of degree 0. (Thanks to Doug Shaw)
90. Page 476, under Figure 12.5: It should be "planar" not "non-planar". (Thanks to Michael Barrus.)
91. Page 477, line 21: It should be "and hence a 5-coloring is possible." (Thanks to Cristina Ballantine)
92. Page 478, line 2 below Figure 12.6: The reference should be to Theorem 12.3.1 (not to Lemma 12.1.1 which doesn't exist!). (Thanks to Cristina Ballantine)
93. Page 479, line 1: It should be y_2 (not x_2). Thanks to Cristina Ballantine)
94. Page 479, Line 2: The reference should be to Theorem 12.3.1 not Lemma 12.1.1. (Thanks to Michael Barrus)
95. Page 503, Exercise 58: The answer to part (a) given on page 593 is incorrect if n is congruent to 3 mod 4. If $n = 4k + 3$, then the answer gives $2k + 1$, but it is impossible to have a regular graph of odd degree $d = 2k + 1$ if the number $n = 4k + 3$ of vertices is odd. In this case the answer should be $2k + 2$. (Thanks to Doug Shaw)
96. Page 548, line 15: Replace G_C with G_E . (Thanks to Balázs Boros.)
97. Page 548, lines -12 and -7: Replace $\rho_n = \iota, \rho, \dots, \rho_n^{n-1}$ with $\rho_n = \iota, \rho_n \dots, \rho_n^{n-1}$. (Thanks to Balázs Boros.)
98. Page 559, line 15: Replace "As shown in Corollary 11.8.8" with "As shown in Corollary 13.1.8". (Thanks to Balázs Boros.)
99. Page 561, line 5 and line -2: Replace "Section 3.1" with "Section 14.1". (Thanks to Balázs Boros.)
100. Page 567, middle of the page in $P_{D_4}(6, 6, 6, 6)$: Replace: 26^3 with 2×6^3 and 36^2 with 3×6^2 . (Thanks to Balázs Boros.)
101. Page 570, lines 8 and 12: $\mathcal{C}(p, q)$ should be $\mathcal{C}_{p,q}$ (Thanks to Reza Kahke-shani)

102. Page 571, line -8: There is a missing right paranthesis at the end of the expression. (Thanks to Reza Kahkeshani)
103. Page 572, line 10: $P_{D_4}(3, 3, 3)$ should be $P_{D_4}(3, 3, 3, 3)$. (Thanks to Reza Kahkeshani)
104. Page 574, line 9: Change $6((r^2+b^2)^3)$ to $6(r^2+b^2)^3$ inside the expression. (Thanks to Reza Kahkeshani)
105. Page 575, line 3: Replace “The set E of edges” with “The set E_1 of edges”. (Thanks to Balázs Boros.)
106. Page 580, Exercise 47: The answer on page 595 has a typo; the z_1^{10} should be z_1^9 . (Thanks to Zhaochen Wang)
107. Page 594, line -2: Change Theorem 13.2.3 to Theorem 14.2.3. (Thanks to Reza Kahkeshani)
108. Page 595, line 10: Replace k with p in the expression. (Thanks to Reza Kahkeshani)