MATH 475; EXAM # 1, 100 points, October 20, 2005 (R.A.Brualdi)

TOTAL SCORE (100 points possible):

Name:

1. (10 points) There are 50 A's, 50 B's, 50 C's, and 50 D's in a hat. If I pick a letter at random out of the hat every second, how long before I am **guaranteed** of having 15 identical letters?

$4 \times 14 + 1 = 57$

2. (10 points) Construct a permutation of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ with inversion sequence $3, 4, 4, 0, 2, 2, 1, 1, 0$.

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3. (20 points) Which binary 9-tuple immediately precedes and immediately follows the 10-tuple $0, 1, 1, 0, 0, 1, 1, 1, 0, 1$ in each of the base 2 generating scheme order of all binary 10-tuples, reflected Gray code order for all binary 10-tuples, and lexicographic order of binary 10-tuples with 5 1's?

Scheme	Immediately **Precedes**	Immediately **Follows**
Base 2 Gen. Scheme	0 1 1 0 0 1 1 1 0 0	0 1 1 0 0 1 1 1 1 0
Reflected Gray Code	0 1 1 0 0 1 1 1 1 1	0 1 1 0 0 1 1 1 0 0
Lex. Order with 6 1's	0 1 1 0 0 1 1 1 1 0	0 1 1 0 0 1 1 0 1 1

4. (15 points) (a) What does $r(3, 4) = 9$ mean?

If the segments of K_9 are colored red or green then one is **guaranteed** that either there is a red K_3 or a blue K_4. It is possible to color the segments of K_8 red or green creating neither a red K_3 nor a blue K_4.

(b) How many ways are there to separate 25 **identical** coins into 5 piles so that each pile contains at least 2 coins?

Number of solutions in nonegative integers of $y_1 + y_\cdot 2 + y_3 = y_4 + y_5 = 15$ and so $\binom{19}{4}$

5. (30 points) Eighteen (18) different students are to be partitioned into 4 teams: the red team, the blue team, the green team, and the orange team.

(a) How many different partitionss are possible if the size of each team can be anything from 0 to 18?

4^{18}

(b) How many different partitions are possible if the red team contains 5 students, the blue team contains 3 students, the green team contains 6 students, and the orange team contains 4 students?

18!/(5!3!6!4!)

(c) Now suppose that the 18 students are to be lined in in a red row, a blue row, a green row, and an orange row. How many ways are there to do this?

(This is like problem Page 81, # 45 (c))

21!/3!

6. (15 points) Give a combinatorial proof of

$$\sum_{k=2}^{n} k(k-1)\binom{n}{k} = n(n-1)2^{n-2}.$$

RHS counts the number of committees (of size at least 2) from n people in which each committee has a pres. (n choices), a vice-pres. different from the pres. ($n-1$ choices) filling out the committee (2^{n-2}) choices.

LHS counts the same thing by first choosing a committee of size k from 2 to n, and then designating one member as pres. (k choices) and a different one as vice-pres. (k-1) choices