MATH 475; EXAM # 2, 100 points, November 23, 2004 (R.A.Brualdi)

TOTAL SCORE (5 problems; 100 points possible):

Do NOT multiply out binomial coefficients and factorials.

Name:

1. [20 points] (a) Use Pascal's formula to verify the identity:

$$\binom{10}{0} + \binom{11}{1} + \binom{12}{2} + \binom{13}{3} = \binom{14}{4}.$$

(b) Now verify the identity by showing that each side counts the same thing. (Hint: Let $X = \{a, b, c, d, \ldots\}$ be a set of 14 elements.)

2. [20 points] (a) Compute the 6th derangement number D_6 in any way you can.

(b) Determine the number of ways to place 6 identical non-attacking roots on the 6-by-6 board below with forbidden positions:

X	X	X			
X	X	X			
			X	X	
			X	X	

.

3. [20 points] (a) Write the exponential generating function for the number h_n of ways to color the squares of a 1-by-n board with colors R, B, G, and Y so that color R occurs an even number of times, color G occurs an odd number of times, and color Y occurs at least once.

(b) Use generating functions to solve the recurrence relation

$$h_n = 2h_{n-1} + 3h_{n-2}, n \geq 0 \text{ where } h_0 = 1, h_1 = 2.$$

4. [20 points] (a) A sequence of numbers $h_n, n \geq 0$ where h_n is a polynomial of degree 3 has difference table looking like:

$$
\begin{array}{ccccccccc}
* & & * & & 2 & & * & & * \\
& * & & -2 & & * & & * & \\
& & * & & 3 & & * & & * \\
& & & * & & 2 & & * & \\
& & & & * & & * & & *
\end{array}
$$

What polynomial is h_n?

(b) Find a formula for $\sum_{n=0}^{m} h_n$.

5. [20 points] Evaluate the Stirling number $S(5,3)$ of the second kind.

(b) What does $S(5,3)$ count?