MATH 475; EXAM # 1, 100 points, October 11, 2007 (R.A.Brualdi)

TOTAL SCORE :

Name: These R. Solutions

Do not multiply out factorials and combination numbers. Circle your answers.

1. (10 points) What should player I's first move be in the following game of NIM in order that she not lose: Piles of 23, 22, and 12.

$23 = 10111$
$22 = 10110$
$12 = 01100$

Adding base 2 without carrying, we get 01101. So we **must** remove 11 from the 12-pile to leave 1 (in base 2, 0001) to balance the game and be assured of winning.

2. (10 points) The Overture Center has performance halls with seat capacities of 1200, 800, 600, 400, and 200 (I am making this up!). How many people must attend a performance on a given night in order that at least one of the performance halls is more than half full?

$600 + 400 + 300 + 200 + 100 + 1 = 1601$

2. **A series of counting problems** (10 points each)

(a) In how many ways can 10 women and 4 men line up in a straight line so that no two men are consecutive?

Line up the woman, choose places for the men, arrange the men:

$10!\binom{11}{4}4!$

(b) In how many ways can 10 women and 4 men arrange themselves in a circle so that no two men are consecutive?

Arrange the women in a circle with one particular woman at the head, choose the places for the men, arrange the men:

$9!\binom{10}{4}4!$

(c) In how many ways can 8 nonattacking rooks be placed on a 12 by 12 board if no rook can be placed in the upper left corner?

Consider all the ways and subtract those with a rook in the upper left corner:

$\binom{12}{8}^2 8! - \binom{11}{7}^2 7!$

(d) 10 identical bagels and 2 identical donuts are devoured at breakfast by 4 people. If everyone eats at least one item and the donuts are eaten by different people, how many possibilities are there for the breakfast to occur?

Give the two donuts to two of the people, then distribute the bagels:

$\binom{4}{2}$ times the number of nonnegative integral solutions of $y_1 + y_1 + y_3 + y_4 = 8$, so

$\binom{4}{2} \times \binom{11}{8}$

(e) 50 people are to be divided (partitioned) into 5 teams of 10 players each. In how many ways can this be done if each team has a different name (Eagles, Bears, Lions, Cubs, and Tigers)? Choose 10 players for the Eagless, then 10 for the Bears, then ...:

$50!/(10!)^5$

In how many ways can this be done if the teams don't have identifiers?

Divide by 5!, the number of ways to assign the names to 5 teams:

$50!/5!(10!)^5$

(g) A swimming contest takes place with 3 swimmers. If ties are allowed (even all finishing at the same time), in how many ways can the race end?

Possibe types of finish: 1,1,1; 1,1,2: 1, 2, 2; 1, 2, 3. Number of each is: 1, 3, 3, 3!, so $1 + 3 + 3 + 6 = 13$

4. (10 points) Consider the subset of size 5 of $\{1, 2, \ldots, 9\}$ given by its bit sequence of length 9: 100110011.

(a) What bit sequence follows it in the base 2 generating scheme for all bit sequences of length 9?

100110100

(b) What bit sequence precedes it in the reflected Gray code scheme? Odd number of 1's.

So it came from an even number of 1's: 100110010

(c) What bit sequence follows it in the lexicographic order for bit sequences with exactly 5 1's?

100101110

5. (10 points) Using $r(3,3) = 6$ and the SPHP, prove that $r(3,4) \leq 12$.

Take 12 points with the line segments joining them colored Red or Blue. Distinguish one of the points, and look at the 11 others joined to it by line segments. Of the 11 line segments, 6 are R or 6 are B. If 6 R, and there is a R-segment between them we have a Red K_3; otherwise all are B, and we certainly have a Blue K_4. On the other hand if 6 are B, then since $r(3,3) = 6$, we either have a Red K_3 among these 6 points, in which case we are done; or we have a Blue K_3 among these 6 points and so a Blue K_4 among the original 12 points. (One could use a similar argument to prove that $r(3,4) \leq 10$.)