MATH 340; EXAM # 1, 100 points, October 11, 2007 (R.A.Brualdi)

TOTAL SCORE :

Name: These R. Solutions

1. (12 points; 4 points each) Let A be an m by n matrix with rows $\alpha_1, \alpha_2, \ldots, \alpha_m$ and let B be a matrix with columns $\beta_1, \beta_2, \ldots, \beta_n$. Then,

- The **entry** of AB in position (i, j) is:

$$\alpha_i \cdot \beta_j$$

- The **rows** of AB are:

$$\alpha_1 \cdot B, \ldots, \alpha_m \cdot B$$

- The **columns** of AB are:

$$A\beta_1, \ldots, A\beta_n$$

2. (8 points) The augmented matrix of a system $Ax = b$ of 4 equations in 5 unknowns x_1, x_2, x_3, x_4, x_5 has rref (reduced row echelon form) equal to

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Is the system $Ax = b$ **consistent**? If so, what is the **solution set**?

The systems is consistent. The solution set is given by:
$x_1 = 2 + x_2 - x_5$
$x_2 = r$ arbitrary
$x_3 = -4 - 3x_5$
$x_4 = 6 - 5x_5$
$x_5 = s$ arbitrary

3. (8 points; 4 points each) Let A be a matrix of size 3 by 4.

1. What **elementary matrix** E has the property that AE is obtained from A by adding 3 times column 2 to column 4?

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. What is the **inverse** of E?

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. (8 points) Let $Ax = b$ be a nonhomogeneous system of m equations in n unknowns, and let $x = u$ be a solution of this system. Let $x = v$ be a solution of the homogeneous system $Ax = 0$ with the same coefficient matrix A. **Prove that, for every scalar k, $u + kv$ is a solution of the homogeneous system $Ax = b$.**

We have $A(u + kv) = Au + A(kv) = Au + k(Av) = b + k(0) = b + 0 = b$.

5. (40 points; 4 points each) Given that A and B are nonsingular matrices of the same order with inverses A^{-1} and B^{-1}, respectively, with determinants $\det A = 3$ and $\det B = 5$. Either give the **answer** to each of the following questions or say that with the given information, it is not possible to say what the answer is (abbreviate: **not possible**):

(a) The inverse of AB is: $B^{-1}A^{-1}$

(b) The inverse of $A + B$ is: NP

(c) The inverse of A^T is: $(A^{-1})^T$

(d) The solution set of $Ax = 0$ is: only the trivial solution $x = 0$.

(e) The solution set of $Ax = b$ is: only $A^{-1}b$

(f) The inverse of A^2B^3 is: $(B^{-1})^3(A^{-1})^2$

(g) The rref of A is: I_n

(h) The determinant of AB^{-1} is: $\det(A)/\det(B) = 3/5$.

(i) The determinant of $A + B$ is: NP

(j) The inverse of the adjoint of A is: $\frac{1}{3}A$

6. (8 points) Let A be a nonsingular matrix of order n. **Prove that the inverse of A is unique**; that is, prove that if B and C are matrices such that

$$AB = BA = I_n \text{ and } AC = CA = I_n,$$

then $B = C$.

NOTE: One cannot use A^{-1} for that it tantamount to assuming that the inverse of A is unique.

So we have:

$B = BI_n = B(AC) = (BA)C = I_nC = C$

7. (8 points) Evaluate the determinant of the following matrix

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 6 & 0 & 7 \\ 0 & 8 & 0 & 0 & 9 \end{bmatrix}.$$

Using the definiiton of the determinant, we see that there are only 2 **nonzero** terms:
$1 \cdot 4 \cdot 5 \cdot 7 \cdot 8$ corresponding to the odd permutation $1, 3, 4, 5, 2$ and
$2 \cdot 3 \cdot 5 \cdot 6 \cdot 9$ corresponding to the even permutation $2, 1, 4, 3, 5$
So determinant is $-1120 + 1620 = 500$

8. (8 points) A matrix A of order 5 has $\det A = 10$ and the submatrix $M_{2,5}$ of A obtained by deleting row 2 and column 5 has $\det M_{2,5}$ =3. Which **entry** of A^{-1} is now determined and what is the **value** of that entry?

The entry in position (5,2) and it is

$$(-1)^{2+5}\frac{3}{10} = -\frac{3}{10}$$