MATH 240; EXAM # 2, 100 points, April 14, 2003 (R.A.Brualdi)

TOTAL SCORE (9 problems):

Name:

Disc. (circle) TUES. 8:50 TUES 12:05 THURS. 8:50 THURS. 12:05

In counting problems you need not evaluate factorials of binomial; coefficients.

1. [16 points] For each of the following relations R on a set A, **circle all that apply**.

(i) $A = \{a, b, c, d\}$ and $R = \{(a, a), (a, b), (b, a), (c, d), (d, d)\}$.

reflexive symmetric anti-symmetric transitive.

(ii) A equal to the set of students in this class and R defined by xRy if and only if x and y have the same hometown.

reflexive symmetric anti-symmetric transitive.

(iii) The relation on a set of 4 elements whose matrix is

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

reflexive symmetric anti-symmetric transitive.

(iv) The relation on a set of 4 elements whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

reflexive symmetric anti-symmetric transitive.

2. [28 points] Solve the following counting problems:

(a) The number of passwords of 5 distinct characters, each equal to one of $0, 1, 2, \ldots, 9, A, B, C, \ldots, Z$ where the last character is a digit.

(b) The number of poker hands of 6 (**note, not 5**) cards from an ordinary deck of 52 cards which consist of exactly three pairs (no three of a kind or 4 of a kind). (Each card in an ordinary deck has one of 13 ranks and one of 4 suits; a pair is two cards of the same rank.)

(c) The probability that in 10 rolls of a die (outcomes are 1, 2, 3, 4, 5, or, 6 dots), we get at least one 6.

(d) The number of ways 21 different computers can be distributed to 5 different rooms if each room is to get at least 4 computers.

3. [10 points] Compute the following:

(a) The coefficient of u^5v^7 in the expansion of $(3u-2v)^{12}$?

(b) $\binom{n}{k}$ if $\binom{n}{k-1} = 4642$, $\binom{n-1}{k-1} = 3265$, $\binom{n-1}{k} = 4326$, $\binom{n-1}{k+1} = 5344$.

4. [12 points] How many ways are there to form a bag of groceries of 15 items where each item is either a carrot, an apple, a bottle of soy sauce, a pie, or a can of coke, if you want to have a least 2 pies and at least 1 coke, and at most one bottle of soy sauce?

5. [12 points]

(a) Let S be a program segment, and let p and q be statements.

(a) If $p\{S\}q$ and $p\{S\}r$, then what other partial correctness statement can you make?

(b) If $p\{S\}r$ and $q\{S\}r$, then the strongest partial correctness statement you can make is:

(b) Determine a recurrence relation, with initial conditions, for the number a_n of bit strings of length n that do contain 2 consecutive 0's.

6. [10 points] Solve the recurrence relation

$$a_n = 8a_{n-1} - 16a_{n-2}, (n \geq 2) \text{ where } a_0 = 3, a_1 = -8.$$

7. [12 points] Determine the number of permutations of the letters B,I,G,R,E,D,W,O,N that do not contain any of the words BIG, RED, WON.