MATH 240; FINAL EXAM, 150 points, 20 December, 2005 (R.A.Brualdi)

TOTAL SCORE (10 problems; plus one 15 point Bonus):

Name:

TA: Anders Hendrickson (circle time) Mon 12:05 Mon 1:20 Wed 12:05 Wed. 1:20

Do NOT compute factorials or binomial coefficients.

1. [10 points] Let A be a set of 7 elements and let B be a set of 9 elements. What are:

- The number of **injective** (one-to-one) functions f from A to B:

- The number of **binary relations** R from A to B:

2. [10 points] Let $A = \{a, b, c\}$. Draw the **diagram** of the partially ordered $(S, \subseteq)$ where S is the power set of A. Then **topologically sort** the elements of this partially ordered set.

3. [15 points] Twenty-four people are to be transported by 6 cars (a Toyota, a Subaru, a Ford, a Jeep, a Chevrolet, and a Chrysler) with 4 people per car.

- In **how many ways** can the transportation be arranged?

- If one person in each car is to have a designated driver, **how many ways** can the transportation be arranged?

4. [10 points] For $n \geq 1$, Let h_n denote the number of ways for a person to climb a flight of n stairs when the person takes 1, 3, or 4 steps at a time. What is a **recurrence relation for h_n with initial conditions**?

5. [15 points] The transitive closure of a binary relation R on a set of 7 elements $\{1, 2, 3, 4, 5, 6, 7\}$ is computed by Warshall's algorithm producing matrices $W_0, W_1, W_2.W_3.W_4, W_5, W_6, W_7$.

- Give a **formula** for computing the entries $w_{ij}^{[4]}$ of W_4 from the entries $w_{ij}^{[3]}$ of W_3:

- If (unspecified entries are 0)

$$W_3 = \begin{bmatrix} 1 & & & 1 & & & \\ 1 & 1 & & 1 & & 1 & \\ & & 1 & & & & 1 \\ & 1 & & 1 & & 1 & \\ & 1 & & & & & \\ & & 1 & & & & 1 \\ & & & 1 & 1 & & \end{bmatrix}$$

, determine W_4 (use the same matrix diagram).

6. [30 points] Consider the binary relations R on a set A defined below.

- What are the **three defining properties** for R to be an equivalence relation?

- What are the **three defining properties** for R to be a partial order?

- Let R be the relation on $A = \{-3, -2, -1, 0, 1, 2, 3\}$ defined by aRb provided $|a| = |b|$ (absolute value). Is R an

  equivalence relation, total order, partial order, or none of these? (circle one)

  If an equivalence relation, how many different equivalence classes are there and what are they?

- Let R be the relation on $A = \{1, 2, 3, \ldots, 20\}$ defined by aRb provided that a is a divisor of b. Is R an

  equivalence relation, total order, partial order, or none of these? (circle one).

  If an equivalence relation, how many different equivalence classes are there and what are they?

- Let R be the relation on the set $A = \{1, 2, 3, \ldots, 19, 20\}$ of 20 elements defined by aRb provided that $a \equiv b$ (mod) 6. Is R an

  equivalence relation, total order, partial order, or none of these? (circle one)

  If an equivalence relation, how many different equivalence classes are there and what are they?

7. [10 points] Let A be a set of 4 elements, and let R be a binary relation on A.

   - If R is a symmetric relation and M_R has 0's and 1's as shown: $\left[\begin{array}{c|c|c|c} 0 & 1 & & 0 \\ \hline & 1 & 0 & 1 \\ \hline 0 & & 1 & \\ \hline & & 1 & 0 \end{array}\right]$, then

     show what the other entries of M_R are.

   - Let M_R have 0's and 1's in positions as shown: $\left[\begin{array}{c|c|c|c} 1 & 1 & & \\ \hline & 0 & 0 & \\ \hline & 1 & & \\ \hline 1 & & & 1 \end{array}\right]$. The **number of ways**

     to complete this to an anti-symmetric, binary relation on A equals:

8. [10 points] Consider the n-cube graphs Q_n:

- What are the **degrees** of the vertices of Q_n?

- Calculate the **number of edges** of Q_n.

9. [20 points] Consider the poset $(S, \leq)$ whose diagram is given below.

Determine

- All **maximal** elements.

- All **minimal** elements.

- All the **upper bounds** of a and b, and then the **LUB** of a and b if it exists.

- All the **lower bounds** of a and b, and then the **GLB** of a and b if it exists.

10. [20 points]

- The following is the postfix (postorder) form of a logical expression. What is its (unambiguous) **usual form**?

$$p\ q\ r \lor \land\ q\ p \land\ \rightarrow\ p\ r \land \lor$$

- What is the **prefix (preorder) form** of the algebraic expression

$$((x \times y) + z) - ((z \times (u + v)) + ((y \times z) \times (x + y)))$$

Bonus Problem (3+2+2+8=15 points)

- Give a recursive definition of a Full Binary Tree (FBT).

- Give a recursive definition of the height $h(T)$ of a FBT T.

- Give a recursive definition of the number $n(T)$ of vertices of a FBT T.

Please Turn Over

- Prove by structural induction that $n(T) \leq 2^{h(T)+1} - 1$ a FBT T.