MATH 240; EXAM # 1, 100 points, February 28, 2003 (R.A.Brualdi)

TOTAL SCORE (11 problems):

Name:

Disc. (circle) TUES. 8:50 TUES 12:05 THURS. 8:50 THURS. 12:05

1. [5 points] Which of the following propositions doesn't belong in this group:

(a) If p, then q.

(b) If $\neg q$, then $\neg p$.

(c) q is a sufficient condition for p.

(d) $\neg p \vee q$.

Circle your answer

2. [6 points] State the converse and contrapositive of the following proposition
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Converse:

Contrapositive:

3. [12 points] Let $P(m,n)$ be the predicate: m *divides* n, where the universe of discourse for each of m and n is the set $Z^+ = \{1, 2, 3, 4, \ldots\}$ of positive integers. Determine the truth values of each of the following propositions:

(a) $\forall\, m\, \forall n\ P(m,n)$.

(b) $\exists m\, \forall n\ P(m,n)$.

(c) $\exists n\, \forall m\ P(m,n)$.

(d) $\forall n\, \exists m P(m,n)$.

4. [9 points] Let $P(x)$ and $Q(x)$ be predicates where the universe of discourse for x is some set U. Let $A = \{x : P(x) \text{ is true}\}$ and $B = \{x : Q(x) \text{ is true}\}$ be the truth sets of $P(x)$ and $Q(x)$, respectively. In terms of A and B give:

(a) the truth set of the predicate $P(x) \rightarrow Q(x)$.

(b) the truth set of the predicate $P(x) \wedge \neg Q(x)$.

(c) Give a predicate, obtained using only $\neg$ (negation) and $\wedge$ (conjunction) from $P(x)$ and $Q(x)$, whose truth set is $\overline{A} \cup \overline{B}$

5. [12 points] Determine whether the following functions are **injective**, **surjective**, or both (**bijective**). **If bijective give the inverse function.**

(a) $f : Z \rightarrow Z$ defined by $f(n) = n + 1$. **Inverse?**

(b) $f : Z^+ \rightarrow Z^+$ defined by $f(n) = 3n + 5$. $(Z^+ = \{1, 2, 3, 4, \ldots, \})$. **Inverse?**

(c) $f : R \rightarrow Z$ defined by $f(x) = \lfloor x + 1/2 \rfloor$. ($R$ is the set of real numbers.) **Inverse?**

(d) $f : R \rightarrow R$ defined by $f(x) = 5x - 2$. **Inverse?**

6. [10 points] Compute the following sum:

$$\sum_{i=0}^{n}\sum_{j=0}^{m} 4^i \cdot 7^j.$$

7. [6 points] Give as good a big-O estimate as possible for each of the following functions:

(a) $\frac{3x^4+2x^3-2x+5}{7x^2+3x+4}$:

(b) $(x\log x + x^2)(x^4 + 5x^2 + 3x)$

8. [8 points] Give the disjunctive normal form (sum of minterms) and conjunctive normal form (product of maxterms) of the Boolean function f of three variables x, y, z which equals 1 if and only if an odd number of the variables x, y, z equals 1.

Disjunctive normal form:

Conjunctive normal form:

9. [10 points] Give a indirect proof of: *If* $a_1, a_2, \ldots, a_n$ *are numbers whose average is 100, then at least one of the numbers is greater than or equal to 100.*

10. [10 points] Prove by mathematical induction: **For all** $n \geq 1$**,** $2^{2n} \equiv 1$ **(mod 3)**.
Be sure to clearly specify the two steps in induction.

11. [12 points] Bob wants to receive a secure message from Alice using the RSA system. He chooses a public key of $n = 221$ by choosing the two primes 17 and 13 whose product is 221. Part of Bob's public key is the encryption exponent $e = 11$ chosen to be relatively prime to $16 \cdot 12 = 192$.

(a) Determine Bob's private decryption key d by using the method of the Euclidean division algorithm.

(b) Using Bob's public keys $n = 221$ and $e = 11$, Alice sends Bob an integer between 1 and 220 which is received by Bob as 60. What integer did Alice send Bob?