Math 114 (Algebra & Trigonometry) NAME:
Lec. 2, Fall Sem. 2001-02

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Solutions of Exam 3

1. If u is between $3\pi/2$ and 2π and $\cos u = 0.3$, determine

(i) $\sin u$ **exactly**: We have $\sin^2 u = 1 - \cos^2 u$. Since u is in the 4th quadrant, $\sin u$ is negative. So

$$\sin u = -\sqrt{1-\cos^2 u} = -\sqrt{1-(0.3)^2} = -\sqrt{0.91}.$$

(ii) $\sin(u/2)$ **exactly**: The angle $u/2$ is in the 2nd quadrant. So

$$\sin(u/2) = +\sqrt{(1-\cos u)/2} = +\sqrt{(1-0.3)/2} = +\sqrt{0.35}.$$

(iii) $\csc(\pi/2+u)$ **exactly**: We have

$$\csc(\pi/2+u) = \frac{1}{\sin(\pi/2+u)} = \frac{1}{\cos u} = \frac{1}{0.3}.$$

2. Find all solution of:

$$\sin x + 1 = \cos x.$$

Squaring both sides and using identity 1., we get,

$$\sin^2 x + 2\sin x + 1 = \cos^2 x = 1 - \sin^2 x.$$

Simplifying this gives $\sin^2 x + \sin x = \sin x(\sin x + 1) = 0$. Thus either $\sin x = 0$, that is, $x = 0 + 2k\pi$ or $\pi + 2k\pi$ (k any integer), or $\sin x + 1 = 0$, equivalently, $\sin x = -1$, that is $x = 3\pi/2 + 2k\pi$. Checking we find that 0 and $3\pi/2$ are solutions of the original equation, but π is not. So the answer is

$$x = 0 + 2k\pi \text{ or } 3\pi/2 + 2k\pi, (k \text{ any integer.})$$

3. Two trains from Chicago are bound for Madison and St. Louis, respectively. At a certain time, the Madison train has traveled 75 miles, and the St. Louis train has traveled 125 miles. If Chicago to Madison is 30 degrees **counterclockwise from north**, and Chicago to St. Louis is 45 degrees **clockwise from south**, how far apart are the two trains at this time?

Drawing a picture (always do this!) and using the law of cosines, we get that the answer is

$$a^2 = 75^2 + 125^2 - 2(75)(125)\cos 105^o$$

which is about 26102, and so a is about 161.56.

4. (i) (a) The **domain** of the inverse sine function is: $[-1, 1]$,
and the **range** is: $[-\pi/2, \pi/2]$.
(b) The **domain** of the inverse tangent function is: $(-\infty, \infty)$,
and the **range** is: $(-\pi/2, \pi/2)$.
(c) Sketch the graph of the inverse sine function:
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(ii) Write the complex number $-4-4i$ in **trigonometric form** with angle argument between 0 and 2π and then compute its **square roots**, that is, find all solutions of $w^2 = -4 - 4i$ in **simplest form**.

The complex number $z = -4-4i$ is in the 3rd quadrant. Its modulus is $\sqrt{(-4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$. Its argument is a third quadrant angle whose reference angle has tangent equal to $-4/(-4) = 1$. Thus the reference angle is $\pi/4$ and the argument of z is $5\pi/4$:

$$z = 4\sqrt{2}(\cos 5\pi/4 + i \sin 5\pi/4).$$

The square roots are

$$\sqrt{4\sqrt{2}}(\cos 5\pi/8 + i \sin 5\pi/8), \text{ and } \sqrt{4\sqrt{2}}(\cos 13\pi/8 + i \sin 13\pi/8).$$

5. Verify to be an identity:

$$\frac{\sin 2u}{1 + \cos 2u} = \frac{\sin u}{\cos u}.$$

Using the double angle formula obtained from the addition formula (identity 3.) and identity 1., we get

$$\frac{\sin 2u}{1 + \cos 2u} = \frac{2 \sin u \cos u}{1 + \cos^2 u - \sin^2 u} = \frac{2 \sin u \cos u}{2 \cos^2 u} = \frac{\sin u}{\cos u}.$$